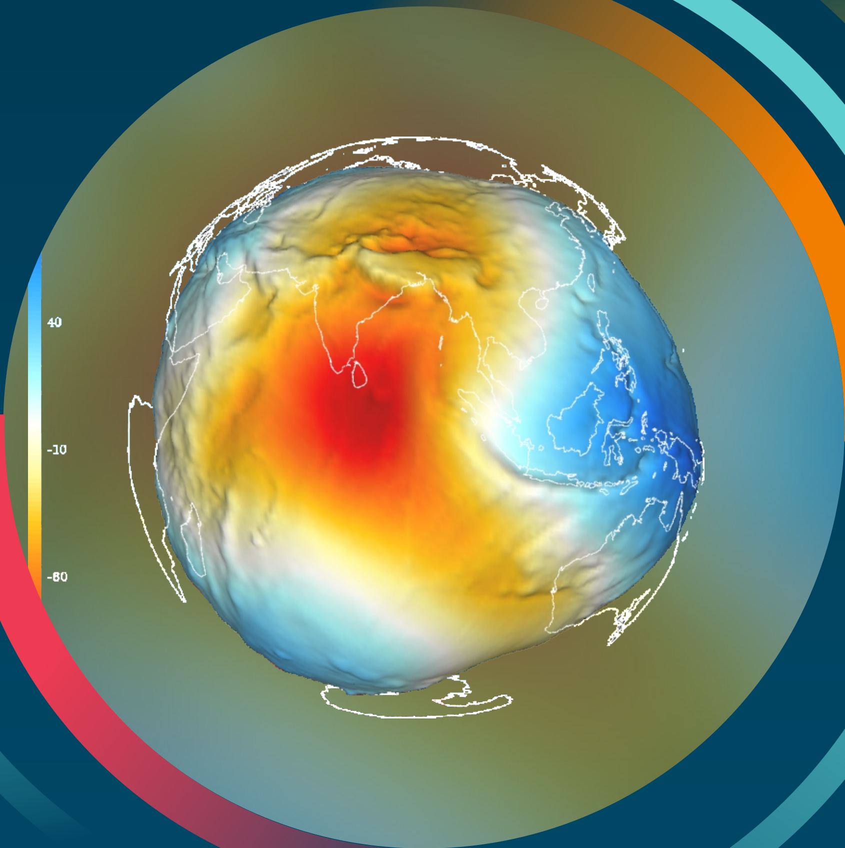
Technical Paper No.: 01/2020

Development of Geoid Model

For Uttar Pradesh & Part of Uttarakhand Under National Hydrology Project (NHP)



S. V. SINGH, DIRECTOR
Geodetic & Research Branch,
Survey of India

Published under the direction of Lt Gen Girish Kumar, VSM (Retd), Surveyor General of India
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FOREWORD

During the last few decades, we have witnessed the widespread adoption of GNSS technology in various applications with an vibrant range of accuracy requirements. However, while GNSS technology has almost replaced all other Survey techniques in measurement of Horizontal position, i.e. Easting-Northing or Lat-Long, accurate vertical positioning, is still a challenge for surveyors. GNSS measures position of any point with reference to a 3D smooth ellipsoidal surface, which approximate shape of earth. For all practical purposes the Earth is not actually smooth like these idealized ellipsoidal models. Due to inconsistent density of the planet, gravitational forces vary over the surface of earth, causing equi-potential surfaces (surfaces over which water do not flow), push out or pull in at different places. High accuracy engineering and scientific works uses height differences as measure of change in potential energy. Therefore to account for push and pulls in equi-potential surfaces, traditional leveling work is used for measurement vertical positions above reference equi-potential surface i.e. Geoid.

The difference between the height above Geoid (orthometric height) and the GPS observed ellipsoidal height is called the geoid height or undulation and Geoid model is a set of mathematical relations between orthometric heights and GPS observed ellipsoidal height, over a part of Earth or for the whole earth. If the geoid undulation is known everywhere, then the GPS height can be simply converted into orthometric height by subtracting the geoid height.

The demand for a modern high resolution Geoid model has grown substantially during the last few years especially after inception of Airborne LiDAR surveys and Drone Surveys. Modern optical and LiDAR sensors are integrated with highly accurate IMU and GNSS devices, which had reduced requirement of Ground Control in Airborne surveys. With a single Base GNSS receiver in area surveyed by aircraft/drone, these sensors can produce High resolution ellipsoidal height point cloud of many square kilometers of land in single flying. However to convert these ellipsoidal heights to Orhtometic heights, Geoid model is required. In India the present-day nationwide Geoid was computed a long time back and which was based on astro-geodetic observations with respect to Everest spheroid. It has various limitations and do not have any significance as far as GNSS solutions for orthometric height, is concerned. Global Earth Gravity Models, such as EGM08, are accurate to few decimetres and sometimes it may be significantly higher in areas of high gravity anomalies, therefore can't be considered for many scientific and engineering applications. So, high resolution and accurate local geoid models are still necessary for the most practical purposes.

In 2016, Survey of India has undertaken the task of generation of High Resolution DEM for in part of catchment areas of the River Ganga and its tributaries under National Hydrology Project (NHP). This work acted as trigger for development of High resolution Geoid Model.

Under guidance of Shri Nitin Joshi, Director and Shri Rajiv Kumar Shrivastava, Director, first attempt on High Resolution Geoid model was made in 2016 for Western Parts of India, which was further expanded for Pan India level By Dr. S. K. Singh, Director, Shri Rajiv Kumar Shrivastava, Director, Shri Neeraj Gurjar, Deputy Director and Shri Bhaskar Sharma, OS and Beta version of Pan India Geoid Model with sub meter accuracy has been developed by G&RB.

With focus on 10 cm accuracy, and learning of past attempts, a detailed plan for State of Uttar Pradesh and Uttarakhand has been prepared and a team of dedicated Surveyors and Officers form Satellite Geodesy wing, High Precision Levelling and Geo-physical Wing of G&RB, carried out High Precision leveling, GNSS and Gravity Observations during field season 2017 – 2018 under NHP Project. The accuracy was all-important because DEM generated by using it, will not only be used in surveying and mapping purposes but also for estimation of surface runoff and planning and design of Hydrological structures on river system. Collection, Compilation and processing of huge amount of data, gathered by field teams, was gigantic task, which has been successfully performed by Geophysical wing of Geodetic and Research Branch.

Lt Gen Girish Kumar, VSM (Retd), Surveyor General of India was constant source of support and guidance in this endeavor. His valuable inputs and backing have made possible to complete this logistically challenging work, within targeted time period.

Major contribution in compiling details of this endeavour and shaping it in form of a book are from Shri Misal Roshan Srivastava, SS, Shri Bhaskar Sharma, OS and Shri Raman Verma, OS.

(S.V. Singh)
Director, G&RB
and Project Director, NHP

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EXECUTIVE SUMMARY

Till the end of 20th century, Mean Sea Level (MSL) was the only reference surface available for height and was determined practically using sea level observations at ports over a certain period of time. For all developmental and engineering projects, MSL based heights have been used in India and most of the countries across world. For this, height above MSL have been provided to Bench Marks (BMs) spread all over the country by means of sprit levelling technique. The levelling throws height to a *point* from a known BM whose height above MSL is known.

But such heights are geometric in nature and don't bear any physical characteristics. It means that such Heights don't follow laws of physics. In simple terms, such height system doesn't always guarantee the fulfillment of water flow criterion i.e. water will always flow from higher potential point to lower potential point. For example, in such geometric height system, it cannot be always ensured that water will flow from higher *'height'* towards the lower *'height'*. Such anomalies occur due the uneven shape & density of the Mother Earth. It is due this very fact that paradox seems to exist. *A point apparently higher than other may have lower potential than the other.*

It is in this brief background that the ***physically meaningful height (which is also called Orthometric Height)*** of a point should refer to the ***Geoid which is an equipotential surface of the earth's gravity field*** and closely approximates the Mean Sea Level (MSL) in *global sense* neglecting the long-term effect of Sea Surface Topography (SST). However, practically, it is very difficult to establish such a surface due to the complexities involved. MSL surface closely approximates the Geoid, which is the actual reference surface for height, by definition.

In order to shift from the geometric height system to orthometric height system that is in order to make the Geoid as reference surface for height instead of MSL, Survey of India (SoI) has redefined its vertical reference surface or Vertical Datum under the project Redefinition of Vertical Datum (RIVD) for heights. This new vertical datum is called Indian Vertical Datum 2009 (IVD2009) which will provide solution to most of the issues/problems related to vertical datum definition.

Apart from being a physically meaningful height system, a Geoid also serves as a very important and foundational surface in today's era of modern surveying. With the advent of satellite based positioning, the modern survey techniques like Drone/Satellite/LiDAR based surveying gives the horizontal position (x, y) of a point which are being computed highly accurately using the Global Navigation Satellite System (GNSS) technology. This technology have been used for more than a decade now but for vertical positioning, still the levelling lines are to be run. ***It is so due to the fact that GNSS technology is based on a reference ellipsoid (WGS84) which is a regular, imaginary and mathematical surface depicting closest possible fit with the Earth or one can say with Geoid which renders the direct use of GNSS height unfit from all practical purposes.***

But the task of spirit levelling, *despite being the most accurate method of throwing heights*, comes with many major disadvantages like time consuming, expensive, require considerable amount of manpower, money and other resources, route dependent (it may be possible to obtained different heights for the same point if levelling lines are run through different routes due to inherent characteristics of level surfaces).

Above constraints have been major hindrances in commissioning the developmental projects of national importance in a time bound manner resulting cost overrun ***because any infrastructural development are being founded on the bed rock of robust, accurate and authentic survey works including the provision of height in area of development***. It happened because there was no means to use GNSS technology for providing height to a point *since both the horizontal and vertical positions refer to different reference surfaces, the ellipsoid and MSL/Geoid respectively*.

So in view of above technical constraints and to mitigate the fallouts of Spirit levelling work, the Geoid model facilitates the conversion of the ellipsoidal height (h^{GNSS}) obtained from GNSS technique to the officially published physical meaningful heights (H) in the region of interest using the relationship,

$$\mathbf{H = h^{GNSS} - N,}$$

where, 'N' is known as Geoid height or Geoid undulation derived from the Geoid model.

To keep up the pace with the geodetic advancements of other countries and with the advancement in the methods of surveying and to maintain the high quality of the height data, Survey of India through its specialized directorate 'Geodetic and Research Branch' have been constantly & tirelessly working towards the development & subsequent amendments of Geoid Model of India.

SoI has computed its 1st Geoid model for India, INDGEOID2018(BETA) covering the entire country with varying accuracy. The accuracy of the INDGEOID2018 is estimated to be better than 8cm within the region of 21°-31° latitude and 71°-83° longitude (mention confidence interval), covering almost 40% of the country which is sufficient for most of the surveying and mapping applications.

After INDGEOID2018, the Geoid for area under National Hydrology Project was developed. The extent of the NHP Geoid developed under NHP is 21°-31° latitude and 71°-83° longitude and within the territorial jurisdiction of Republic of India. The accuracy of the NHP Geoid is estimated to be better than 10 cm within the above mentioned NHP area.

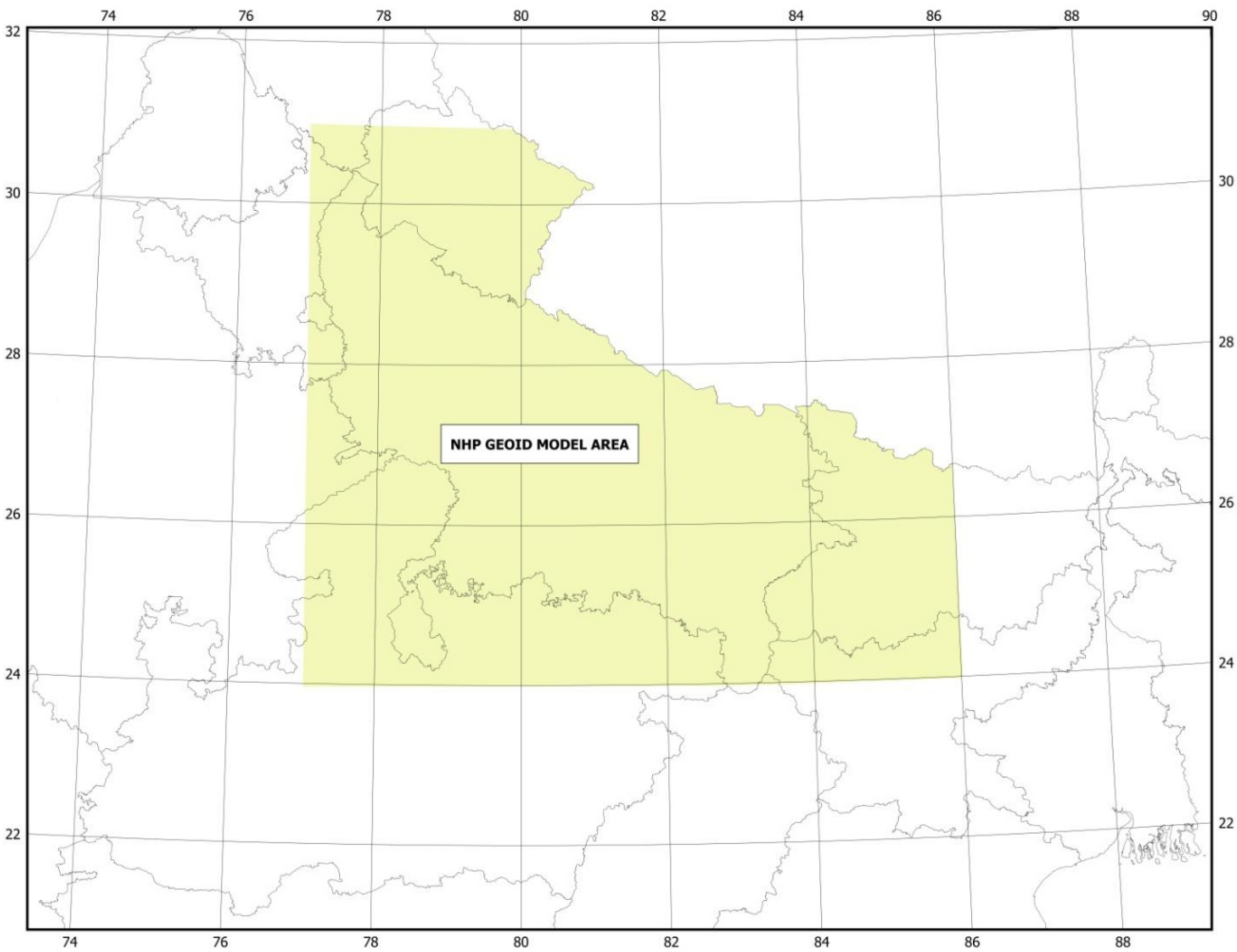


Fig. A-NHP Geoid Model Area

In continuation of the NHP project, the Geoid under NMCG and remaining NHP area is under development. The extent of the Geoid developed under NHP cum NMCG is:

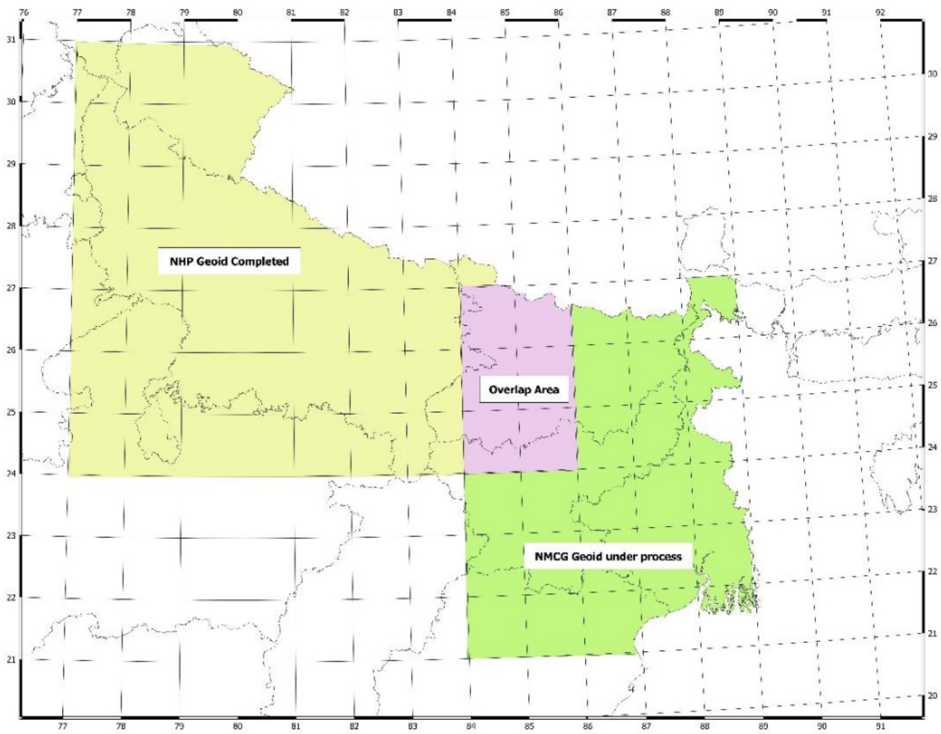


Fig. B- Area for Geoid under NHP & NMCG

The improvement of accuracy of the 'Geoid' over the NHP area mentioned above was follows:

Geoid	No of check points	RMSE (m)	Std. Dev. (m)	Mean (m)
NHP Geoid	151	0.077	0.076	-0.009

Table A- Improvement of Accuracy due to Development of NHP Geoid

The Geoid model will be made available to the user community subject to the policy decision to be taken by the government in this regard.

SoI constantly strive to upgrade its Geoid model and it is in the process of acquiring more and more gravity and GNSS/levelling data to refine the existing version of Geoid model in order to improve the magnitude of accuracy beyond the above-mentioned region of the country. Data from NMCG project and field work of Kerala is under scrutiny and the subsequent amendment of the Geoid model will be carried out in due time.

As & when the gravity and GNSS/Levelling data is made available a refined version of this model will be computed and published. The heights provided by this model will refer to recently published Indian Vertical Datum 2009 (IVD2009).

This has been a major achievement of Survey of India in recent years. With the availability revised and refined High resolution Geoid model users can provide horizontal and vertical coordinates of a point simultaneously using the GNSS technology without resorting to tedious and cumbersome technique of levelling. This will help in minimizing the requirement of time, money and manpower resources to a greater extent thereby reducing the cost of survey manifolds. ***Survey of India has taken Geoid model development as a continuous activity and planned to develop more precise versions of Geoid models for the entire country in near future.***

PURPOSE

The purpose has been to develop Geoid model for Uttar Pradesh and part of Uttarakhand. Geoid model provide Geoid heights that can be combined with GPS-derived ellipsoid heights to produce values consistent with the official vertical datum published orthometric heights under IVD-2009. Hybrid Geoid models are created by constraining a gravimetric Geoid model to published heights using GPS observations on levelled bench marks.

A hybrid Geoid model requires two datasets:

1. A purely gravimetric Geoid model, which is created from a number of terrestrial, airborne, and space-based gravimetric datasets
2. A network of passive bench marks like SBMs or GCPs or any other permanent monument where both the ellipsoid height from GPS and the orthometric height from geodetic levelling are observed.

Combining these two datasets gives the hybrid Geoid model the positive attributes from each individual dataset. The gravimetric Geoid models are considerably accurate over long distances (or wavelengths). The gravimetric Geoid is also a continuous surface with no gaps, allowing areas where no levelling lines are present to be accurately modeled. The GPS on bench mark data, where they exist, accurately reflect the shape of the Geoid and provide higher resolutions over small geographic regions. The fusion of these datasets results in a hybrid that is both seamless and accurate at all distances.

The following sections focus on the methodology, input datasets, analysis, and performance of various Geoid models published till date by SOI and associated products:

- Methodology
- Input Datasets
    - Gravimetric Geoid Models
    - Gravity observations: mesh work & on levelling lines
    - Geoidal Undulation on Bench Marks
- Results
- Performance Analysis
    - Omission Error Analysis of various datasets
    - Commission Error Analysis of various datasets

The following sections provide technical information, but avoid detailed background or explanations of the mathematical or geophysical concepts and try to cover possible scenarios for usage of Geoid Model developed under NHP.

CHAPTER 1: FUNDAMENTALS

1.1. INTRODUCTION

To define a point on Earth, a reference system is needed which can be used to realize stable datum. The geodetic reference system in each country has been defined as two separate datum i.e. horizontal and vertical datum.

With regard to vertical datum, the primary reference for heights is the '**Geoid'** which may be defined as:

*"The equipotential surface of the Earth's gravity field which corresponds most closely with '**mean sea level (MSL)**' in the 'open oceans', ignoring the semi-dynamic effects of ocean currents. It extends continuously through the continents and, as such, is commonly used as the datum for topographic elevations in many countries"* [w.e Featherstone, jf Kirby sjclaessens, m Kuhn 2017]

It is important to briefly explain the concept behind the realization of a MSL as a close substitute to a Geoid.

1.1.1. Mean Sea Level:

In general, in absence of a high resolution local Geoid, vertical datum is established through the observation of local MSL at a number of tide gauge sites over a sufficient long period of years. Sea level is the base level for measuring elevation and depth on Earth. Because the ocean is one continuous body of water, its surface tends to seek the same level throughout the world. However, winds, currents, river discharges, and variations in gravity and temperature prevent the sea surface from being truly level. So that the surface of the ocean can be used as a base for measuring elevations, the concept of "local mean sea level" has been developed. In general, local mean sea level is determined by taking hourly measurements of sea levels over a period of 19 years at various locations, and then averaging all of the measurements.

The 19-year period is called a Metonic cycle. It enables scientists to account for the long-term variations in the moon's orbit. It also averages out the effects of local weather and oceanographic conditions. The effects of tides and of other periodic variation can be filtered out using appropriate numerical filters. The result is supposed to yield the MSL at a given site.

To establish a vertical network origin, it is only necessary to define this point to have an elevation zero. That "zero" elevation definition is then usually transported with high precision levelling techniques via closed polygons for the rest of the country (Pina et. al. 2001).

In India the vertical datum was defined in 1909 using the MSL data furnished from nine tide gauge states at Karachi, Bombay, Karwar, Beypore, Cochin, Nagapattinam, Madras, Visakhapatnam and False Point (Burrard, 1910). The datum defined in 1909 is still in use and suffice most of the practical applications.

1.1.2. Need for Geoid as Vertical Datum:

With the advent of the Global Positioning System (GPS), the survey methodology as well as the way of thinking in geodesy has changed drastically. This revolution, apart from core surveying, has extended to mapping, navigation and Geographic Information System (GIS) fields, etc. The modern airborne survey techniques like drone based photogrammetric survey, LiDAR based topographical mapping, etc. also uses GPS in various modes for navigation as well as for geo-referencing of images captured. The ability of GPS to derive very precise and accurate horizontal positions is well known and fully established.

But since GPS is based on WGS84 ellipsoid, it becomes apparent that the ability to derive precise, and perhaps more importantly accurate and meaningful elevation with respect to a physically meaningful datum is the most difficult component to accomplish from GPS technology. Thus, the biggest drawback in GPS is the vertical component offered from this technique. The ellipsoidal heights derived from GPS observations do not have any physical meaning unless it is transformed to levelled heights of local reference datum based on water flow criteria. Thus the satellite techniques necessitate the requirements of a precise model of Geoid at global, regional and local scales in the practical world (engineering surveys) as well as for the geodetic applications. This will complement the horizontal positioning directly obtained from GPS. Huge quantum of manpower, resources and time will be saved.

Due to this reason the determination of Geoid as a vertical datum for height measurement has been one of the main interests of Geodesists during the last few decades. The Geoid model not only enables the users to convert GPS ellipsoidal heights to leveled heights (orthometric or normal heights) but also plays an important role in combining levelling data with GPS measurements to study the vertical crustal movements for a longer period of time (Kuroishi et. al, 2002).

The development of improved high order global geopotential models, such as EGM2008, etc. and the advancement of theory and methodology in Geoid determination have made progress in accurate Geoid modeling studies possible in many parts of the world. The current (i.e. CHAMP, GRACE and GOCE) dedicated satellite missions have been instrumental in increasing the levels of accuracy for the determination of long to-medium-wavelength components of Earth's gravity field spectrum. The global geopotential models constructed from data derived from these satellite missions have improved the existing global geopotential models, notably in the low and medium frequencies. The development made in theoretical and practical aspects of Geoid determination over the last few years enables users to produce detailed local Geoid models with spatial resolution of *one or a few kilometers* where dense data on surface gravity and topography are available.

1.1.3. Relationship between Ellipsoidal & Orthometric Height:

Gravimetric methods of Geoid determination with dense data on surface gravity and topography and a global geopotential model are widely applied to model the Geoid regionally and locally. The resulting models recover the short wavelengths effect at regional and local scale but suffer from systematic errors in longer wavelengths due to errors of the global geopotential model and procedures as well as datum inconsistencies of local levelling networks (Kostakis and Sideris, 1999). Combination of GPS, Levelling and Gravimetric Geoid information has been a key procedure for various geodetic applications. Although these three types of height information are considerably different, mainly in terms of physical meaning, yet reference surface definition / realization, observational methods and accuracy may fulfill the simple geometrical relationship (Kostakis and Sideris, 1999)

$$N = h - H + \epsilon \quad (1.1)$$

Where h is ellipsoidal height obtained from GPS observations, H is orthometric height derived from spirit levelling, N is Geoid height and Є is small quantity due to deflection of the vertical and the curvature of plumb line (Torge, 1980).

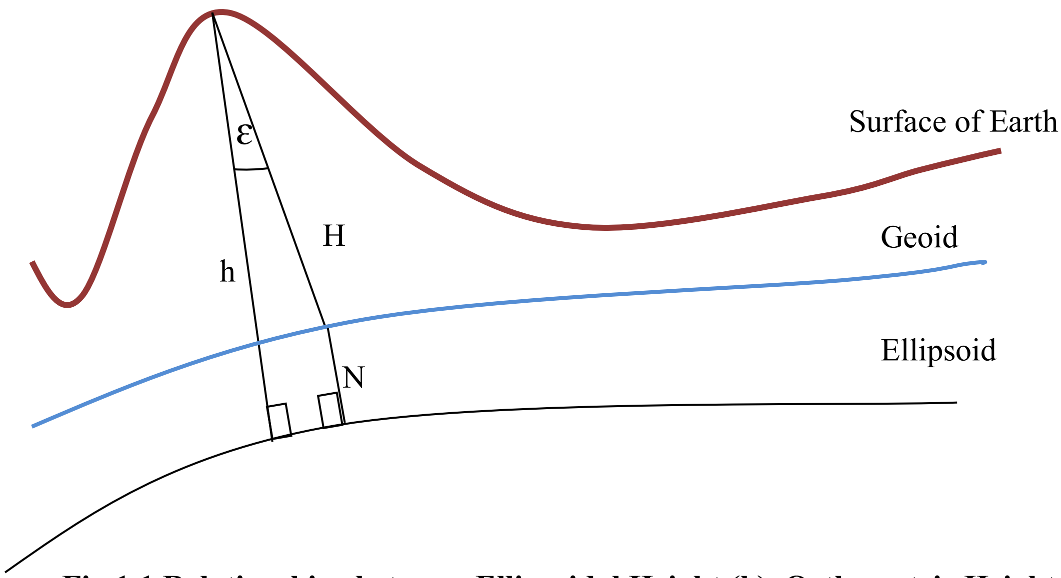


Fig 1.1 Relationships between Ellipsoidal Height (h), Orthometric Height (H), Geoidal Height (N) and Deflection of Vertical (ε).

1.2. AN INSIGHT INTO GEOID:

A Geoid is nothing but an imaginary surface around the globe which is an equipotential surface. That is, it is surface on which every point has equal gravitational potential. This equipotential surface best approximates the MSL over the entire Earth. It has been defined as the datum for the orthometric height system.

Geometrically, the Geoid is irregular in shape, and therefore does not allow the mathematical computation of the horizontal position of points. That is such irregular surface can't be described using simple mathematical equations. Therefore, a reference surface of regular shape usually a biaxial ellipsoid is selected to define the horizontal positions. The geometrical relationship between the Geoid and the reference ellipsoid surface can be fully described by the offsets, called Geoidal separation (N) and a small correction ε.

However, the correction term ε is neglected for most of the practical applications. The Geoidal separation is measured along the ellipsoidal normal, whereas the orthometric heights are measured along the curved plumb line.

The determination of orthometric heights is closely allied to engineering works concerned with the control and flow of water. Water obeys the law of physics and flows “downhill” from one equipotential surface to another. In fact, a “free” body of water forms its own equipotential surface. *This natural occurrence is the reason why an equipotential surface is the most sensible datum for heights*. It also provides the reason for most countries adopt some form of MSL as the datum for heights, since all water bodies of the Earth are discharged to the ocean.

However, today it is well known that this is not strictly correct definition of Geoid, as *mean sea level departs from the equipotential surface by up to two meters* due to various oceanographic phenomena, such as variable temperature, salinity, sea surface topography etc. (Fotopoulos, 2003).

Due to vastness of areal extent of the mother Earth and non-feasibility of observations on each & every point of the Earth, the Geoidal separation value (N) cannot be determined, exactly at every point of the Earth’s surface, but instead must be interpolated from a predefined Geoid models locally, which facilitates the possibility of rapid height determination using GPS. Therefore, as parallel to the wide range of applications of GPS technique, a demand for a high precision Geoid model referring to a global geocentric datum has appeared. As the result of this, a precise Geoid model has become an important component in configuring a geodetic infrastructure globally or locally.

1.3. GEOID MODELLING – VARIOUS APPROACHES:

Geoid modeling processes are variable in nature, and can be broadly classified in following categories depending upon the area covered viz. global, regional and local.

i. Gravimetric approach mainly applied for global and regional models (Featherstone, 2001; Chen and Yang 2001, Kuroishi et al. 2002). In this approach, the basic inputs are gravity measurements, terrain model of the area and a global geopotential model (GGM), and Geoid is determined using widely accepted technique termed as remove-compute-restore method (Featherstone et. al, 1998, Yang and Chen 1999, Chen and Yang 2001, Hipkin et. al. 2004).

ii. Geometric approach for determination of local/regional Geoid with an aim to replace leveling measurements with GPS surveys (Seker and Yildirim, 2002, Tscherning et. al 2001, Kavzoglu and Saka, 2005, Nagarajan and Singh, 2005). The geometric approach envisages the determination of Geoid in a relatively small area by a combination of GPS derived ellipsoidal heights and leveling heights at some reference points, called bench mark. Geoid heights (N) are calculated according to the basic equation (1.1), while it can be interpolated analytically or graphically at any other GPS observed point. This procedure can be performed by transformation, point wise interpolation or surface fitting methods and ascribing the Geoid as analytical surface. A plane or low order polynomial is usually used to model the Geoid surface (Featherstone et. al. 1998). The geometric method has been mostly preferred in practical application of geodesy, i.e., for large scale map production, engineering projects etc

iii. A combination of above two techniques commonly known as hybrid approach of Geoid determination.

Whereas, the gravimetric approach in various forms of computational procedure has been the most popular during the past two decades, the geometric approach has started gaining popularity in recent past due to improvement in ellipsoidal height determination by GPS. It is due to improved orbit information availability in terms of precise ephemeris and an equally vibrant range of data processing software e.g. BERNESE, GAMIT, etc.

1.4. GENERAL PRINCIPLES & FUNDAMENTALS

1.4.1. Earth's Gravity Field

Let us assume that point masses are distributed continuously over the entire body of the Earth having the density function ρ,(x,y,z) then the integral:

$$V(P) = V(x, y, z) = G \iiint\limits_{Earth} \frac{\rho(Q)}{l} dv_Q$$

V(P) represent the gravitational potential where ρ(x,y,z) is an external point Q lies within the Earth body which forms the centre of the volume element dv_Q and l is the distance between P and Q. The potential V is continuous throughout the whole space and vanishes as l tends to infinity. This can be seen from the fact that for very large distances l, the body acts approximately like a point mass with the result eq. (1.2) may be expressed as

$$V = \frac{GM}{r} + \frac{0(l)}{r^2} \qquad r \to \infty \tag{1.3}$$

The total force acting on the body at rest on the Earth's surface is the resultant of gravitational force and centrifugal force of the Earth's rotation. The gravity potential W is the sum of V and potential of the centrifugal force:

$$V_c = \frac{1}{2} W^2 (x^2 + y^2) \tag{1.4}$$

so that $W(x, y, z) = V(x, y, z) + \frac{1}{2} W^2 (x^2 + y^2)$ (1.5)

ω being the angular velocity of the Earth's rotation, which is considered as constant (Moritz 1980). The field of potential V is called the gravitational field and the field of potential W is the gravity field. The gravity vector g is the gradient of w:

$$g = grad\, W = \left(\frac{\partial W}{\partial x}, \frac{\partial W}{\partial y}, \frac{\partial W}{\partial z}\right) \tag{1.6}$$

The direction of the gravity vector is the direction of plumb line or the vertical. It significance in geodetic and astronomical measurement is well known. Outside the attracting masses, above the Earth's surface S, V satisfies Laplace's equation:

$$\Delta V = 0 \tag{1.7}$$

The solution of this equation is called harmonic function. In the Earth interior, inside S, the potential V satisfies Posisson's equation:

$$\Delta V = -4\,\pi G\rho \qquad (1.8)$$

ΔV and ρ refer to the same point inside S. The corresponding relations for the gravity potential W are:

$$W = 2\,\omega^2 \qquad \text{(outside S)} \qquad (1.9)$$

$$\Delta W = -4\pi G\rho + 2\omega^2 \qquad \text{(inside S)} \qquad (1.10)$$

The surfaces $W(x,y,z) = W_o$ = constant, on which the potential W is a constant, are called equipotential surfaces or level surfaces.

Differentiating the gravity potential W = W (x,y,z), we find:

$$W = \frac{\delta W}{\delta x}dx + \frac{\delta W}{\delta y}dy + \frac{\delta W}{\delta z}dz \qquad (1.11)$$

Using the scalar product eq. (2.24) reads as:

$$dW = grad\,W.dx = g.dx \qquad (1.12)$$

If dx is taken along the equipotential surface, then the potential remains constant and dW = 0 so that

$$g.dx = 0 \qquad (1.13)$$

Therefore, this equation expresses the well-known fact the gravity vector is normal to the equipotential surface passing through the same point.

1.4.2. Spherical Harmonics

If Laplace's equation is expressed as $\Delta V = 0$ in terms of spherical coordinates (r, θ, λ) and attempt to solve it by separating the variables (r, θ, λ). To accomplish this, the gravitational potential V may be expressed as a product of three independent functions.

$$V = f\,(r)\,g\,(\theta)h\,(\lambda) \qquad (1.14)$$

After some mathematical manipulation, the solutions are found to be:

$$f\,(r) = r^n or\, f\,(r) = \frac{1}{r^{n+1}} \qquad (1.15)$$

$$g\,(\theta) = P_{nm}\,(cos\,\theta) \qquad (1.16)$$

$$h\,(\lambda) = cos\,m\lambda \text{ or } h\,(\lambda) = \sin m\lambda \qquad (1.17)$$

Where n is called degree (n = 0, 1, 2, 3--------) and m the order of the functions (m = 0, 1, 2, 3 -----n) under consideration. The function $P_{nm}(\cos\theta)$ is called Legender function, and defined as ($\cos\theta = t$):

$$P_{nm}(t) = \frac{1}{2^n n!}(1-t^2)^{m/2}\frac{d^{n+m}}{dt^{n+m}}(t^2-1)^n \qquad (1.18)$$

For m = 0, the Legendre polynomials is:

$$P_n(t) = P_{n0}(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n \qquad (1.19)$$

For m≠ 0, the P_{nm} (t) is called the associated Legendre function. The polynomial may be obtained by more simpler and computer oriented recursion formula:

$$P_n(t) = -\frac{n-1}{n}P_{n-2}(t) + \frac{2n-1}{n} + P_{n-1}(t) \qquad (1.20)$$

The product of function (1.16) and (1.17) are Legendre surface harmonics

$$R_{nm}(\theta,\lambda) = P_{nm}(cos\,\theta)\,cos\,m\lambda \qquad (1.21)$$

$$S_{nm}(\theta,\lambda) = P_{nm}(cos\,\theta)\,sin\,m\lambda \qquad (1.22)$$

The product of (1.14), (1.16) and (1.17) $r^n R_{nm}(\theta, \lambda)$, $r^n S_{nm}(\theta, \lambda)$, $r^{-(n+1)} R_{nm}(\theta, \lambda)$ and $r^{-(n+1)} S_{nm}(\theta, \lambda)$ are the corresponding solid spherical harmonics. These functions as well as their linear combinations are also harmonics. In particular the series given below may be used for representing the Earth's external gravity potential, which is a harmonic function.

$$V(r,\theta,\lambda) = \sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\sum_{m=0}^{n}P_{nm}(cos\,\theta)(A_{nm}\cos m\lambda + B_{nm}\sin m\lambda) \qquad (1.23)$$

1.4.3. Geodetic Boundary Value Problem

There are several mathematical relationships that can be established between the quantities defined above via Laplace's differential equation for the disturbing gravity potential supplemented by mixed boundary condition of potential theory. This boundary condition represents a link between quantities, which can be derived from observed magnitude of gravity at the topography and unknown values of the disturbing gravity potential at the Geoid.

Consider the expansion of normal potential function U, which is a harmonic function, about the point P in the form of Tailor series (Heiskenen and Moritz, 1967).

$$U_P = U_Q + \left\{\frac{\delta U}{\delta n}\right\}_Q N = U_Q - \Upsilon N \tag{1.24}$$

We have $W_P = U_P + T_P = U_Q - \Upsilon N + T_P$ (1.25)

Since $W_P = U_Q = W_O$

$T_P = \Upsilon N$

Or

$N_P = T_P/\Upsilon$ (1.26)

This is a famous Brun's formula which relates the Geoidal undulation to the disturbing potential.

Again applying the Tailor's theorem to the normal gravity function at P, we have:

$$\Upsilon_P = \Upsilon_Q + \frac{\delta\Upsilon}{\delta h} N \tag{1.27}$$

Now consider the gravity disturbance δg:

$$\delta g = g_p - \Upsilon_p = -\left\{\frac{\delta W}{\delta n} - \frac{\delta U}{\delta n}\right\} = -\frac{\delta T}{\delta n} \tag{1.28}$$

Since elevation h is reckoned along the normal n, we may also write:

$$\delta g = -\frac{\delta T}{\delta h} \tag{1.29}$$

Comparing (1.29), (1.28) and (1.26) we have

$$\frac{\delta T}{\delta h} = g_P - \Upsilon_P = (g_P - \Upsilon_Q) - \frac{\delta\Upsilon}{\delta h} N \tag{1.30}$$

Substituting for $(g_P - \Upsilon_Q)$

$$\Delta g = -\frac{\delta T}{\delta h} + \frac{\delta\Upsilon}{\delta h} N \tag{1.31}$$

$$\frac{\delta T}{\delta h} - \frac{1}{\Upsilon}\frac{\delta\Upsilon}{\delta h} T + \Delta g = 0 \tag{1.32}$$

This expression (eq. 1.32) is called the fundamental equation of physical geodesy, as it relates the measured quantity Δg to the unknown anomalous potential T. It can be used as a boundary condition for determination of T. If Δg is assumed to be known at every point of the Geoid, the linear combination of T and $\frac{dT}{dh}$ define that surface. The determination of T is therefore termed as boundary value problem (GBVP) of potential theory. If it can be solved for T, the Geoidal height N can be easily computed from Brun's formula (Heiskenen and Moritz, 1967).

1.4.4. Solution of GBVP, Stokes Concept

The solution to the GBVP is most expediently sought in the form of spherical harmonic series. The boundary condition (1.32) may be written in the form of spherical approximation as:

$$\frac{\delta T}{\delta r} + \frac{2T}{r} + \Delta g = 0 \tag{1.33}$$

Since in spherical approximation

$$\Upsilon = \frac{GM}{r^2}, \frac{\delta\Upsilon}{\delta h} = \frac{\delta\Upsilon}{\delta r} = -\frac{2GM}{r^3}, \frac{1}{\Upsilon}\frac{\delta\Upsilon}{\delta h} = \frac{-2}{r} \tag{1.34}$$

The anomalous potential T = W- U is harmonic and can be expanded into a series of spherical harmonics:

$$T(r,\theta,\lambda) = \sum_{n=0}^{\infty} \left\{\frac{R}{r}\right\}^{n+1} T_n(\theta,\lambda) \tag{1.35}$$

Also $\frac{\partial T}{\partial r} = \frac{1}{r}\sum_{n=0}^{\infty}(n+1)\left\{\frac{R}{r}\right\}^{n+1} T_n(\theta,\lambda)$ (1.36)

T_n (θ, λ) is Laplace's surface harmonics of degree n. The function T (r, θ, λ) can be expressed on the Geoid, a spherical approximation corresponds to the sphere r = R as:

$$T = T(r,\theta,\lambda) = \sum_{n=0}^{\infty} T_n(\theta,\lambda) \tag{1.37}$$

For gravity anomaly, the boundary condition yields:

$$\Delta g = \frac{1}{r}\sum_{n=0}^{\infty}(n-1)\left\{\frac{R}{r}\right\}^{n+1} T_n(\theta,\lambda) \tag{1.38}$$

On the Geoid r = R, it becomes:

$$\Delta g = \frac{1}{R}\sum_{n=0}^{\infty}(n-1)T_n(\theta,\lambda) \tag{1.39}$$

It may be noted that even if the anomalous potential has first degree term, gravity anomaly will never have it. We may also directly express Δg (θ, λ), as a series of Laplace's surface harmonics:

$$\Delta g(\theta,\lambda) = \sum_{n=0}^{\infty} \Delta g_n(\theta,\lambda) \tag{1.40}$$

n=0

Comparing two equivalent expressions, we get:

$$\Delta g(\theta,\lambda) = \frac{(n-1)}{r} T_n(\theta,\lambda) \sum_{n=0}^{\infty} \Delta g_n(\theta,\lambda) \tag{1.41}$$

So that

$$T = \sum_{n=0}^{\infty} = R\sum_{n=0}^{\infty} \frac{\Delta g_n}{n-1} \tag{1.42}$$

This equation shows again that there should be no first-degree term in the spherical harmonic expansion of Δg, otherwise the terms Δg /n-1 would be infinite for n=1. We shall now assume that harmonics of degrees zero and one are missing. Therefore, the summation is started with n=2.

Now Δg_nmay be written in the form of surface integral:

$$\Delta g_{\mathrm{n}} = \frac{2n+1}{4\pi}\iint_\sigma \Delta g\, P_n(\cos\psi) d\sigma \tag{1.43}$$

Where ψ is angular distance the computation point and variable points on the surface σ, therefore:

$$\mathrm{T} = \frac{R}{4\pi}\sum_{n=2}^{\infty}\frac{2n+1}{n-1}\iint_\sigma \Delta g\, P_n(\cos\psi) d\sigma \tag{1.44}$$

or

$$\mathrm{T} = \frac{R}{4\pi}\iint_\sigma \sum_{n=2}^{\infty}\frac{2n+1}{n-1} P_n(\cos\psi)\Delta g d\sigma \tag{1.45}$$

$$\mathrm{S}(\psi) = \sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi)$$

The function is the expression for Stoke's function in terms of Legendre polynomials. Since it depends on ψ only a closed form of the same reads (Hobson, 1931):

$$\sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi) = \mathrm{S}(\psi) = 1 + \frac{1}{\sin\frac{1}{2}\psi} - 6\sin\frac{1}{2}\psi - 5\cos\psi - 3\cos\psi l_n\left(\frac{1}{2}\psi + sin^2\frac{1}{2}\psi\right) \tag{1.46}$$

We finally get the closed solution to the GBVP in the following form.

$$\mathrm{T} = \frac{R}{4\pi}\iint_\sigma \Delta g\,(\theta,\lambda)\sin\Phi\, d\sigma \tag{1.47}$$

If we assume that the problem of redundant masses is eliminated by some approximate correction to Δg, the disturbing potential can be transformed to Geoidal height simply through Brun's formula:

$$\mathrm{N}(\theta,\lambda) = \frac{R}{4\pi}\iint_\sigma \Delta g\,(\theta,\lambda)\mathrm{S}(\psi)\mathrm{d}\sigma \tag{1.48}$$

This equation is known as Stokes' formula, and is considered to be the most important formula of physical geodesy as makes it possible to determine unknown Geoid height from observed gravity data. Stokes' formula is based on a spherical approximation of the Earth, causing relative errors of the order of $3x10^{-3}$ (Heiskenen and Moritz, 1967) or absolute errors within one metre (Olliver, 1980).

In addition to the assumption that the boundary surface of the Geoid is sphere, there exist some implicit conditions in equation (1.48). First, Stokes' formula represents a global integration where gravity observations are required to be known globally. This condition is an impracticable preposition. Therefore, the integral is usually performed over a spherical cap of limited radius ψ_o, and the remote zones are accounted for by augmenting the local gravity field with a global geopotential model. Secondly, the expression for Stokes' function in terms of Legendre polynomials contains no zero-or first-degree terms. Thus suppressing these harmonics in the calculated disturbing potential. The zero-degree term represents the global mean difference of the mass and potential between the Earth and reference ellipsoid. The first-degree terms are only zero if the center of the reference ellipsoid coincides with the mass center of the Earth (geocentre). Therefore, it is important that the reference ellipsoid is geocentric. Thirdly, no mass outside the Geoid is assumed in order for the disturbing potential to be harmonic. This poses the biggest theoretical and practical obstacle in Stokes' approach for Geoid determination. The presence of topographical masses violates the basic assumption behind this solution, which is the harmonicity of the disturbing potential outside the Geoid. Therefore, the topographic masses outside the Geoid may be removed by suitable gravity reductions (downward continuation).

1.4.5. The Global Geopotential Models (GGM)

One of the most important applications of gravimetric Geoid is to provide fundamental transformation from GPS derived ellipsoidal height to orthometric heights. If this transformation is not applied and only ellipsoidal heights are used, the possibility exists that water may appear to flow the uphill. During the past three decades of satellite geodesy, geodesists around the world have derived a number of GGMs based on purely satellite observations and also following a combined approach using satellite and terrestrial data in Geoid modeling process. A GGM predominantly represents the long and medium wavelength (>100km) component of the Earth's gravity field. Rapp and Rummel (1975), among others provided some of the earliest gravimetric Geoid using a GGM thus avoiding the global gravity data coverage requirement and reducing the time period for Geoid computation significantly. Therefore, GGMs play very important and vital role in determination of gravimetric Geoid at a regional or local scale and also makes the Geoid computation process much easier.

Presently GGMs are used as a routine stage in the procedures employed for computation of gravimetric Geoid. The past three decades has witnessed the development of a series of GGMs of increasing spherical harmonic degree and order and hence resolution. The most recent models are available complete to degree and order 360 and capable of providing long and medium wavelength information of the Earth's gravity field to a resolution of approximately 50 km. Efforts are going on to produce the models of higher degree and order following the launching of dedicated satellite gravity mission e.g. CHAMP, GRACE and GOCE etc.

In order to obtain an optimal solution of gravimetric Geoid to support GPS height determination, it is necessary to select the best fitting geopotential model for the region of interest. This can be achieved by using the following statistical tests:

1. Comparing the Geoid undulation computed from various GGMs with geometrically derived Geoid heights at points within the region of interest. It can be achieved by computing difference of GPS derived ellipsoidal heights and orthometric heights from levelling observations.
2. Comparing gravity anomalies computed from GGM with gravity anomalies derived from the National gravity data base.

The geopotential model that provides the closest statistical fit to GPS/leveling and local gravity data can be considered as the most suitable model for the determination of the gravimetric Geoid. Such verification is important when combining a GGM with Stokes' formula because a best fitting model may reduce the impact of assumptions and approximations inherent to the Stokes' formula.

The RCR technique envisages the role of Global Geopotential Model (GGM) to remove the long wavelength effect from the local gravity data. This is an important part of the process in regional Geoid determination as more than 90% contribution comes from a GGM. Therefore, selection of GGM which is a best fit to the local gravity field is a critical issue, particularly in the present scenario when a number of GGMs are available in the public domain. Best fitting of GGM to a particular regional gravity field reduces the amount of Geoid contribution that must be made by a regional integration of Stokes' formula or some modification there of (Amos and Featherstone, 2003). Many authors (Zhang and Featherstone, 1995, Kirby and Featherstone, 1997, Kearsley and Halloway, 1989, Amos and Featherstone, 2003) have done studies on evaluation of available GGMs to decide their suitability to provide long wavelength contribution to regional gravimetric Geoid. The evaluation of GGMs assumes much more significance today when there is a continuous flux of improved long wavelength Geoid information from the current and planned dedicated satellite gravity field missions (i.e. GRACE, CHAMP and GOCE). In addition several new or refined theories have been proposed for the determination of Geoid.

1.4.6. Use of GGM in Geoid Determination Process

The Earth's gravitational potential could be expressed in terms of an infinite series of spherical harmonics outside the attracting masses of the earth. A GGM represents this gravitational field derived from the satellite and terrestrial observations. Due to lack of availabilities of continuous gravity information and poor resolution of satellite data, these series of harmonics are usually truncated.

The modern geopotential models are available with following components:

- The set of coefficients (C_{nm} and S_{nm}) from degree 2 to degree n and order m,
- The adopted gravity mass constant (GM_G) value used in deriving the model,
- An equatorial scale factor, a_G,and
- The tide system of the model

These components are used to compute the gravity potential W_G (Heiskanen and Moritz, 1967) outside a sphere of radius r = a_G. The subscript "G" refers to the GGM values and subscript "E" are related to the parameter of the geodetic reference frame of interest. The following formula can be used for defining the gravitational component of the potential of a GGM:

$$W_g = \frac{GM_g}{r}\left[1 + \sum_{n=2}^{n_{max}} \sum_{m=0}^{n} \left\{\frac{a_g}{r}\right\}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\bar{P}_n^m \cos\theta\right] \quad (1.49)$$

In principle, the reduction steps presuppose that the density distribution of the topography and the gravity gradient from the Geoid to the topographic surface are known. The topographic mass-shift necessitates the correction for the so-called in direct effect. Theoretically, after the correction, any reduction method may provide the unique Geoid undulation. In practice however, only approximate estimate of mass densities is used and the free-air reduction of normal gravity is frequently applied with each method leading to various degrees of erroneous results. Essentially, each method gives a compensated-Geoid or co-Geoid.

The first serious attempt to remedy this flow in the application of Stokes' method can be attributed to Helmert (1884), which was subsequently named as Helmert's Condensation method. Helmert's Condensation method is regarded as the most straight ward method (Vaniĉek and Martinee, 1994) for treatment of topographic effect in Geoid computation.

Where, $C_{nm}$and S_{nm} are fully normalized spherical harmonic coefficients, of degree n and order m; GM_G is the product of the Newtonian gravitational constant and mass of the GGM; the triplet (r, θ, λ) are spherical polar coordinates, a_G is the equatorial radius of GGM and P_n^m are fully normalized associated legendre function.

The GRS 80 reference ellipsoid which provides the normal gravity field to most of the recently developed GGMs practically coincides with WGS 84, the reference ellipsoid for GPS coordinates system (Seeber, 2003). The WGS 84, after several refinements now coincides with Earth's center of mass at the level of 1 cm (Merrigan et. al, 2002).It fulfill the requirement of geocentric reference ellipsoid as per the assumption made in application of Stokes' formula for Geoid determination. Therefore, the first-degree harmonics (n =1) are inadmissible and the summation in eqn. (2.63)begins at the second degree. However, the zero degree term still remains due to an imprecise knowledge of the product of the Newtonian gravitational constant and Earth's mass.

The gravitational potential of the normal ellipsoid (U_E) can be expressed as harmonic series (Moritze, 1980)

$$U_E = \frac{GM_E}{r}\left[1+\sum_{n=2}^{\infty}\left\{\frac{a_E}{r}\right\}^n (\bar{C}_{n0})\bar{P}_n \ \cos\theta\right] \tag{1.50}$$

Where, $\bar{C}_{n0}$ are fully normalized even zonal harmonics only (i.e. = 2, 4, 6, 8, 10…..); GM_E is the product of Newtonian gravitational constant and mass of the normal ellipsoid; and a_E is the equatorial radius of the normal ellipsoid. The difference between the gravitational potential of GGM from Eq. (2.63) and the normal potential Eq. (2.64) at the same point defines the disturbing potential T_G as:

$$T_g = \frac{GM_G-GM_E}{r} + \frac{GM_G}{r}\sum_{n=2}^{n_{max}}\sum_{m=0}^{n}\left\{\frac{a_G}{r}\right\}^n (\overline{\Delta C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}\cos\theta \tag{1.51}$$

where,

$$\Delta C_{n0} = \overline{\Delta C}_{n0} - \frac{GM_E}{GM_G}\left\{\frac{a_E}{a_G}\right\}\bar{C}_{n0} \tag{1.52}$$

For even zonal harmonics only (Vaniček et. al., 1987), the first term on the right hand side of eq (2.65) is the zero degree term in the disturbing potential due to difference in the estimates of the mass of the Earth used by the GGM and normal ellipsoid. The disturbing potential is more meaningfully expressed as the Geoid height, measured positive away from the Geoidal surface along the ellipsoidal normal. Using the generalized Brun's theorem accounting for the difference in potential between the surfaces of the normal ellipsoid and the Geoid, the Geoid height is given by :

$$N = \frac{T_G}{\gamma} - \frac{W_0 - U_E}{\gamma} \tag{1.53}$$

Where W_o is the gravity potential on the surface of the Geoid and γ is the normal gravity on the surface of the ellipsoid.

Using eq. (2.65) and (2.67) the Geoid height, including the zero-degree term, is expressed in terms of geopotential coefficients as:

$$T_G = \frac{GM_G - GM_E}{r} - \frac{W_0 - U_E}{\gamma} + \frac{GM_G}{r}\sum_{n=2}^{n_{max}}\sum_{m=0}^{n}\left\{\frac{a_G}{r}\right\}^n (\overline{\Delta C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}\cos\theta \tag{1.54}$$

The zero degree term is given by:

$$T_G = \frac{GM_G - GM_E}{r} - \frac{W_0 - U_E}{\gamma} = \delta(\mathrm{GM}) - \delta\mathrm{W} \tag{1.55}$$

The zero-degree term is manifested in the form of shift in position of the Geoid with respect to the geocentre and does not affect the Geoid height differences. Therefore the determination of orthometric height differences from GPS, as used in practical surveys is insensitive to any zero-degree bias in the Geoid (Kirby and Featherstone 1997).

1.4.7. Types of Global Geopotential Models

Presently a number of GGMs are available in public domain derived from satellite only or satellite and terrestrial data combination technique. It is desirable to classify them according to the technique and data used for the model development. The entire range of GGMs available today may be categorized in to three different classes.

1. **Satellite only GGM:** These classes of GGMs are derived from the orbit analysis of satellites. There are various factors associated with satellite observations viz., power decay of the gravitational field with altitude; inability to track the complete orbits from ground based stations; imprecise modelling of atmospheric drag; perturbations due to unknown sources and under-sampling of the global gravity field due to the limited number of satellite orbital inclination available. All these factors affect the quality of gravity data collected by the satellite missions and induce noise in the ultimate GGM solutions (Rummel et. al., 2002). Therefore, even if the satellite GGM are available above degree 100, the higher degree coefficients say greater than 30 are heavily burdened with noise (Fig.2.5). However in recent times, the problem has been redressed by launching dedicated satellite gravity missions viz, CHAMP, GRACE and GOCE etc.

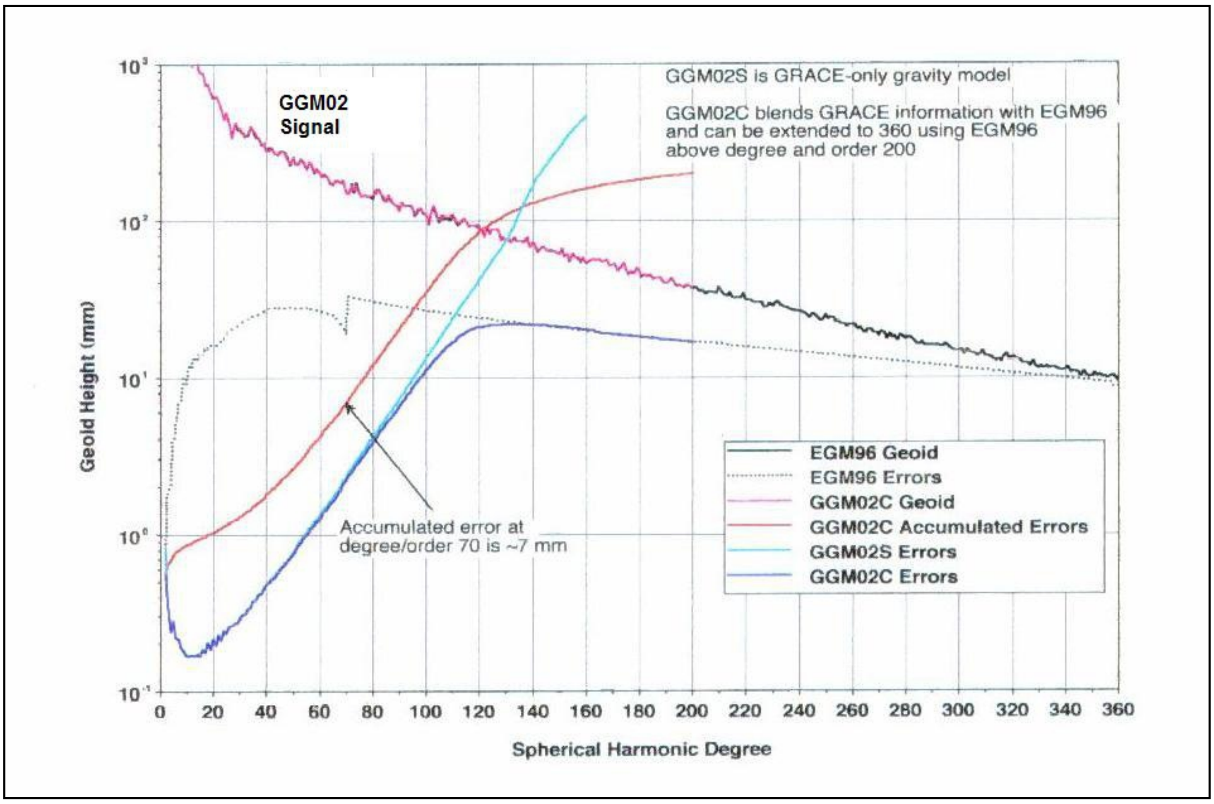


Fig 1.2 Degree signal and error statistics of EGM96 and GRACE Gravity Model(GGM02)(Tapley et.al.,2005)

2. **Combined GGM Solutions:** These classes of models are derived from combination of satellite data, land and ship borne gravity observations and marine gravity anomalies derived from satellite radar altimetry and airborne gravity data (Rapp, 1997 b). This combination allows increasing the maximum spherical harmonic degree of GGM. To overcome the limited precision of satellite only GGM due to above mentioned restrictions as well as the spatial coverage and quality of additional data used in the modeling process, a special band-limited combination method is generally applied in order to preserve the high accuracy from the satellite data in the lower frequency band of the satellite only model and to facilitate a smooth transition to the higher frequency domain with the help of terrestrial data. However distortions and offsets among different vertical geodetic datums cause long-wavelength errors in terrestrial gravity anomalies, among with many other sources (e.g. Heck, 1990). These will generate low frequency errors in the combined GGM if the effect is not properly high pass filtered from the combined solution. The low frequency errors to a certain extent may be filtered out by giving proper attention to the zero- and first-degree terms of Geoid and gravity anomalies in GGM. This will remove the datum distortions effect in computation of a regional Geoid model.

1.5. GRAVIMETRIC TERRAIN REDUCTION METHODS

Almost all the geodetic measurements are reduced to Geoid, and the "Geodetic Boundary Value Problem" is solved for the Geoid by means of classical Stokes' and Molodensky's problems and other modern methods. Although, both the conventional approaches cannot make full use of all types of available gravimetric data, as they are based on a continuous global coverage of gravity anomaly data over the entire Earth's surface but they are of paramount importance for theoretical definition of the Geoid and related quantities. These approaches are quite often used in various modified forms for practical Geoid determination. Other modern approaches viz. altimetry-gravimetry (Wagner, 1979) and Bjerhammar's problems (Bjerhammer, 1964), which can make use of heterogeneous and discrete data are still under developmental stage and have not yet become standard geodetic tools for precise determination of Geoid. Considering these facts and other constraints related to data and software availability, the classical Stokes' formula has been considered most appropriate method for determination of gravimetric Geoid. The advantage of this approach is that the solution obtained by applying this formula leads to the determination of the level surface (Geoid), which is simply defined in terms of physically meaningful and geodetic important potential W. The disadvantage is that the potential W inside the Earth, and hence the Geoid W = constant, depends on the density ρ because of Poisson's equation.

$$\Delta W = -4\pi G\rho + 2\omega^2 \tag{1.56}$$

Therefore, the density of masses at every point between the Geoid and the topography must be known, at least theoretically, in order to determine the Geoid. This is clearly impossible, and therefore to circumvent the likely influence of density variation, though practically insignificance, some assumptions have to be made while using the Stokes' formula for Geoid determination.

1.5.1. Geoid & co-Geoid

Stokes' formula deals with the determination of a potential, harmonic outside the masses, from gravity anomalies, given everywhere on the Earth's surface. Following assumptions are implicitly made in defining the Stokes' integral(Heiskanen and Moritz, 1967):

(i) The Geoid is a boundary that defines the plane surface between Earth's interior and exterior. Consequently, all surface data have to be reduced to this boundary first.

(ii) The disturbing potential T is harmonic on the boundary which implies that there are no masses outside the Geoid.

(iii) Reference ellipsoid,(a)has the same potential Uo = Wo as the Geoid, (b)encloses a mass that is numerically equal to the Earth's mass and (c)has its center at the center of gravity of the Earth.

(iv) Spherical approximation of the normal potential, that is:

$$U = GM/r \tag{1.57}$$

The harmonic condition for disturbing potential leads to the solutions of Laplace's equation:

$$\Delta T = 0 \tag{1.58}$$

The condition of 'no masses outside the Geoid' may be met either shifting the external mass below the Geoid or completely removing it mathematically before applying the Stokes' and related formulae. In both the cases of treating the external mass, mathematically the gravity is bound to change. Also, gravity measurements are done at ground level but gravity is required at sea level. These two phenomena greatly influence the gravity data reduction method in order to obtain the boundary value at the Geoid.

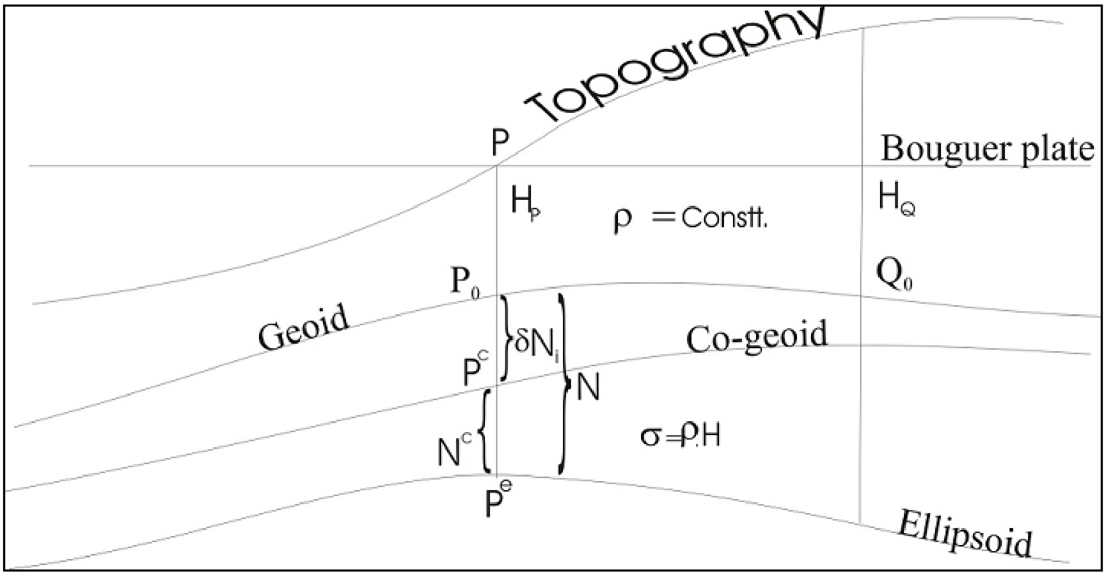


Fig 1.3: Indirect effect due to topography

The Earth and its gravity potential W are approximated by a reference ellipsoid of revolution, usually with the same mass and rotation rate as the Earth, having a normal potential U. The process of removing the external masses mathematically results in change in level surfaces, and hence the Geoid. This is termed as indirect effect and the changed Geoid is called the co-Geoid (Fig. 1.3). The basic principle used in Stokes' method of Geoid determination may be described as under:

(i) Removing or shifting the masses outside the Geoid will affect the gravity value g at P which will be duly taken into the account.

(ii) The station P downward continues to P_o on the Geoid and again this effect is taken into consideration.

(iii) The indirect effect $\delta N_i = P_oP^c$ is determined by applying the Brun's theorem.

$$\delta N_i = \frac{\delta W}{\Upsilon} \qquad (1.59)$$

(iv) The gravity station is now moved from P_o to P^c at Co-Geoid to give boundary value of gravity at the co-Geoid g^c.

(v) The co-Geoid N^c is computed from the reduced gravity anomalies:

$$\Delta g_c = g_c - \Upsilon \tag{1.60}$$

by the Stokes' formula to give $N^c = QP^c$

(vi) Finally the Geoidal undulation N is obtained as:

$$N = N_c + \delta N_i \tag{1.61}$$

In principle, every gravity reduction that provides boundary values at the co-Geoid may be used for Geoid determination process provided the indirect effect is properly taken into account.

1.5.2. Solution to the GBVP

An anomalous potential T^o at every point P_o on the Geoid is defined as given by the expression:

$$T_P^o = W_P^o + U_P^o \tag{1.62}$$

The classical BVP is to determine T^o that satisfies Laplace's equation:

$$\nabla^2 T^o = \frac{\partial^2 T^o}{\partial X^2} + \frac{\partial^2 T^o}{\partial Y^2} + \frac{\partial^2 T^o}{\partial Z^2} \tag{1.63}$$

Where ∇^2 is Laplace's operator under the following boundary condition on Geoid:

$$\frac{\partial T^o}{\partial h} - \frac{1}{\Upsilon}\frac{\partial \Upsilon}{\partial h} T^o + g^o = 0 \tag{1.64}$$

The solution of equation (1.62) under the condition of equation (1.64) provides T^oas a function of the gravity anomalies and is given by the Stokes' integral:

$$T^o = \frac{R}{4\pi}\iint_\sigma \Delta g^o S(\Psi)\, d\sigma \tag{1.65}$$

From Brun's formula

$$N = \frac{R}{4\pi\lambda}\iint_\sigma \Delta g^o S(\Psi)\, d\sigma \tag{1.66}$$

The parameters of this expression have already been explained in the previous section. Stokes' integral when applied to gravity anomalies on the Geoid surface gives the undulation of the Geoid under the' no masses outside' condition. One of the most common and frequently used technique to take care of the topographic masses of density ρ in practical Geoid determination is

Helmert's second method of condensation (Heiskanen and Moritz, 1967, Nagarajan, 1994, Bajracharya, 2003). This method of treating external/masses has been used here which comprises of following steps:

(i) Remove all masses above the Geoid.

(ii) Lower station from P to P_o (Fig. 3.1) using the free-air reduction of gravity (to be discussed later in this section).

(iii) Restore masses condensed on a layer on the Geoid with density $\sigma = \rho.H$

This procedure gives

$$\Delta g_o = \Delta g_P - A_P + F + A_{po}^{c} = \Delta g_P + F + \delta A \qquad (1.67)$$

Where, (Δg_p + F) is the free-air gravity anomaly at P, A_p is the attraction of topography above the Geoid at P and A_{Po}^{c} is the attraction of the condensed topography at P_o. Sideris (1987) has shown that in planar approximation the attraction change δA is equivalent to the classical terrain correction (TC).

1.5.3. Gravity Data Reduction

There are various gravimetric reduction techniques in physical geodesy to remove topographical masses above the Geoid in the classical solution of the geodetic boundary problem using Stokes' formula. In principle, all the reductions scheme are equivalent and may lead to the same result provided the other aspects of data gridding and treatment of indirect effect are dealt properly. However, there are certain requirements that restrict the effectiveness of practically useful reductions. The main requirements are:

1. The reduction must yield gravity anomalies that are small and smooth so that they can be easily interpolated, i.e. a single anomaly value can act as representative as possible of the whole neighborhood.

2. The reduction may correspond to a geophysically meaningful model so that resulting anomalies are also useful for geophysical and geological interpretations.

3. The indirect effect may not be unduly great.

The specific choice of gravity reduction method depends on the magnitude of its indirect effect, the smoothness and magnitude of the resultant gravity anomalies and their associated geophysical interpretation. The complete Bouguer reduction and Topographic-isostatic reductions, viz Airy-Heiskanen and Prat-Hayford etc.(Heiskanen and Moritz,1967) exhibit all the characteristics of a good reduction scheme. These methods introduce large indirect effects which are smaller than those of the Bouguer scheme, but still larger than those exhibited by Helmert's second method of condensation, and thus have not been used in this study for practical Geoid determination.

Vaniĉek, et.al. (2001) have discussed the physical meaning and interpretation of Bouguer plate reduction, and concluded that the classical method of reduction does not imply the removal of the effect of the plate. Rather it is the removal of the upper half of the plate and does not make any physical sense.

For the reasons cited above or otherwise, the Helmert's second method of condensation is by far the most widely used technique of treating the topographical masses gravity data reduction for gravimetric Geoid modeling. The present study has also made use of this method, and is discussed in detail in later sections.

1.5.4. Free-Air Reduction

Much of the geodetic literature describes the process of computing gravity anomalies as a reduction process during which gravity values observed at the surface are reduced to some datum level, usually mean sea level. Theoretically reduction to Geoid requires knowledge of the rate of change of the actual gravity field $\partial g/\partial h$ between the Geoid and Earth's surface (Fig. 1.4). In practice, g is observed on the Earth's surface, then the value g_o at the Geoid may be obtained as a series of Taylor expansion:

$$g^o = g - \frac{\partial g}{\partial h}.H \pm ------ \quad (1.68)$$

Where, H is the elevation of the gravity station above the Geoid. Neglecting the higher order terms:

$$g^o = g + F \quad (1.69)$$

Where F = $-\frac{\partial g}{\partial h}.H$ is the free air reduction to the Geoid. In practice the actual gradient $\frac{\partial g}{\partial h}$ is difficult to measure and downward continuation is unstable problem, therefore , $\frac{\partial g}{\partial h}$ is generally approximated by the theoretically defined gradient of normal gravity, $\frac{\partial \Upsilon}{\partial h}$ This quantity is the familiar free air gradient, normally taken as 0.3086 H mgal.

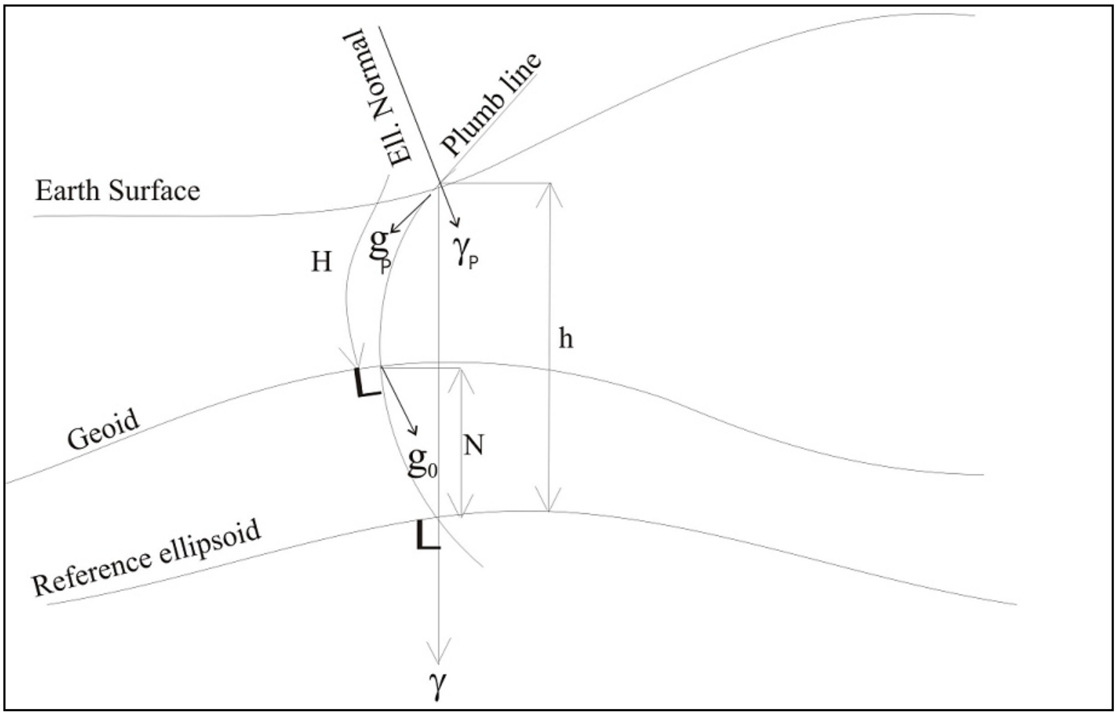


Fig1.4 Parameters used in the definition of gravity anomalies (Hackney and Featherstone, 2003)

Finally using eq. (1.67), the free air gravity anomalies may be given by:

$$\Delta g_{\mathrm{o}}^{\mathrm{FA}} = \Delta g_P - \Upsilon_{\mathrm{o}} - 0.3086\mathrm{H} \quad (1.70)$$

Where, Υ_o is normal gravity defined for reference ellipsoid.

1.5.5. Terrain Correction

The terrain correction is used as part of a “condensation” reduction in Helmert’s second method to replace the gravitational effect of in-situ topographical masses with an equivalent layer situated (condensed/compressed) at the Geoid. Alternatively, the mass can be moved mathematically inside the Geoid (Martinec &Vaniček, 1994). This condensation reduction is required to make the gravity anomaly field a harmonic function, thus permitting the solution of the geodetic boundary-value problem by Stokes’ method. Essentially, the terrain correction is applied to the free-air gravity anomaly to yield the Faye gravity anomaly, which is an approximation of Helmert’s gravity anomaly (Hackney & Featherstone, 2003). The terrain correction is also used during the gridding and prediction of gravity data to reduce aliasing effect prior to computation of Geoid.

1.5.6. Residual Terrain Model (RTM)

In practice, a global high order spherical harmonics reference gravity field is generally used to invoke a Remote-Compute-Restore (RCR) type of estimator in computation of a local/regional gravimetric Geoid. Such a global spherical harmonic expansion includes the effect of the global topography. If a spherical reference field is used i.e. gravity field modelling is applied on the residual:

$$T^r = T - T_{ref} \tag{1.71}$$

Then the subtraction of a further topographic/isostatic effect may introduce long-wavelength effect into the residual potential. To avoid this, it is desirable that only the shorter wavelength of topographic effect may be removed.

This concept may be applied using a residual terrain model (RTM). The RTM scheme envisages the use of a mean elevation surface h_{ref} which is defined by low pass filtering of local terrain heights. A reference density model is implicitly defined which has crustal density (e.g. 2.67 g/cm^3) upto the reference level $h_{ref.}$ This reference surface corresponds to T_{ref} (spherical harmonic expansion of global topography to same degree and order), and topography masses above or below of this surface are removed or filled up respectively. The RTM reduction scheme is illustrated in fig. 1.5

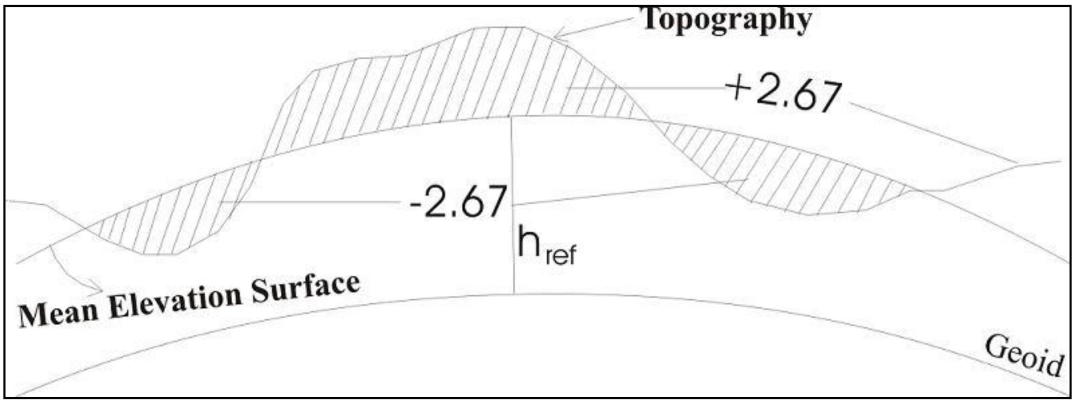


Fig1.5 Principle of RTM Effect(Forsberg,1994)

The topographic RTM density anomalies can make a balance act of positive and negative density anomalies, representing areas where the topography is either above or below the reference topography.

The RTM gravity terrain effect in the planer approximation can be expressed in the following form (Forsberg, 1984):

$$\Delta g_{RTM} = G\rho \int_{-\infty}^{\infty} \int_{z=h_{ref}(x,y)}^{z=h\ \ (x,y)} \int \frac{z-h_P}{\left((x-x_P)^2+(y-\ y_P)^2+(z-\ z_P)^2\right)^{3/2}} dxdydz \quad (1.72)$$

Where h is the topographic height given by DEM. When the mean elevation surface represents a long wavelength surface, the RTM reduction may be approximated by a Bouguer reduction to reference level:

$$\Delta g_{RTM} = 2\pi\rho G\left(h - h_{ref}\right) - c \quad (1.73)$$

The first term of this equation is computed from the thickness of Bouguer plate at passing through the point of computation above the reference elevation, and the second term represents the classical terrain correction.

1.5.7. Practical Considerations in Gravimetric Terrain Correction/Reductions

The free air gravity anomaly is known to be highly correlated with the topography, and if there are rough estimation of topographic masses in the computation, the free air-gravity field will be only an approximation and that is why the prediction could not be successful. For any of the techniques employed for solving the Stokes' integral equation (either Fast Fourier Transform (FFT) or quadrature based summation, a regular geographic grid, of mean gra vity anomalies is required. In order to predict gravity anomalies in these regular configurations from irregularity spaced gravity observations, interpolation and some time extrapolation is also needed.

Since the terrestrial local gravity data provide high frequency addendum to the low frequency Geoid signals from a GGM, the entire phenomenon is to be analyzed in purview of the signal sampling theory (e.g. Brigham 1988). According to the theory the gridding, any type of irregularly sampled data is subjected to a phenomenon called aliasing in the presence of improperly sampled high frequencies. That is, if there are insufficient observations to sample the complete gravity field spectrum, which is the case for most of the developing nation including India, then high frequency signals are incorrectly propagated (i.e. aliased) into the low frequencies. This generates spurious long wavelength terrestrial gravity anomalies that will induce noise in the computed Geoid.

CHAPTER 2: METHODOLOGY

The theoretical and practical developments in gravimetric Geoid computation have undergone a sort of revival and given a twist to the conventional methods over the last few years. This may be attributed to the current level of achievable accuracy of ellipsoidal heights derived from GPS observation, which has reached to the centimeter level as compared to decimeter level, a few years ago. This technological development led to optimize the gravimetric Geoid solution through new ideas of modifying the Stokes' formula by one way or the other in order to yield a better fit between gravimetric Geoid and GPS/levelling undulations. Stokes' original theory is based on several assumptions, which if neglected, may produce an error in Geoidal height of approximately one meter. These comprise:

1. Earth mass equals that of reference ellipsoid.
2. Geoid potential equals ellipsoid surface potential.
3. The ellipsoid is geocentric.
4. The ellipsoid and Geoid both are spherical.
5. Gravity data are available over the entire earth.
6. No masses exist external to Geoid.

In the current state of geodetic knowledge, the first three assumptions are retained as the GRS 80, reference ellipsoid is considered to be best fit to the Earth. Any biases or zero order terms which may induce errors can generally be neglected as the GPS observations are generally performed in relative mode to derive the geodetic quantities. The requirement of gravity coverage over the entire earth cannot be met anyway due to inaccessibility and confidentiality. This unresolvable constraint has precluded on accurate determination of Geoid using Stokes' formula, instead an approximate solution is used in practice. This problem is overcome partly by inclusion of a global geopotential model (GGM) and partly by some modification of Stokes' integral. The GGM offers a superior information source of the low frequency component of Geoid via a set of spherical harmonic coefficients and when combined with local terrestrial gravity data, the medium frequency component of Geoid, gives a reasonable estimate of regional Geoid. Also, due to the density of the gravity data, which on the average is no better than 5' by 5', the short wavelength component will not be present. The high frequency or short wavelength component is computed using topographic heights, which are usually given in the form of a Digital Elevation Model (DEM). These frequency contributions are shown in figure 2.1.

The basic formula for regional Geoid undulations N, by combining a GGM, local terrestrial gravity anomalies Δg, and heights H in a DEM will read:

$$N = N_{GGM} + N_{\Delta g} + N_H + \epsilon_N \tag{2.1}$$

$$\Delta g^M = \Delta g_F - \Delta g_{GGM} - \Delta g_H \tag{2.2}$$

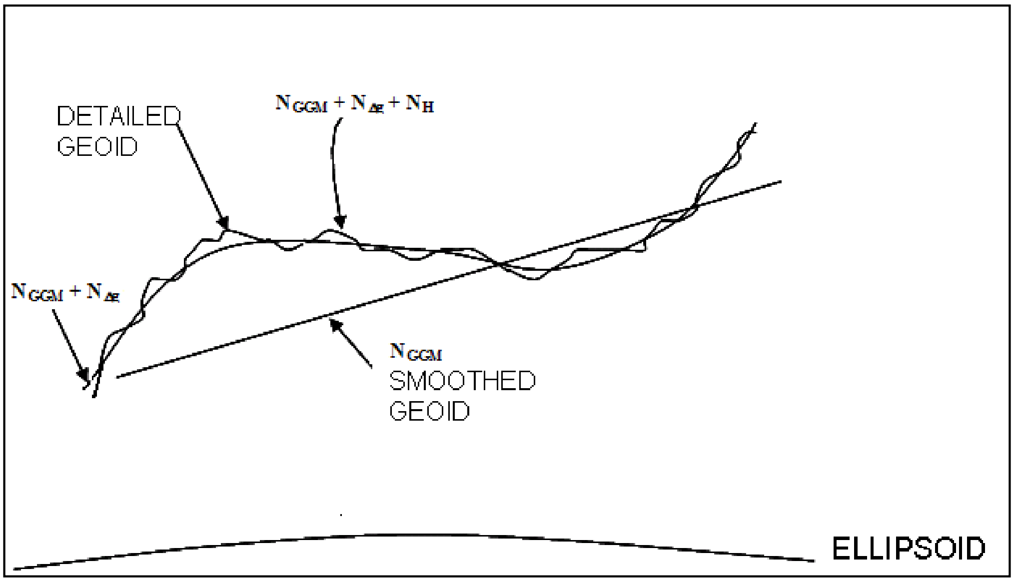


Fig 2.1 Contributions of different data to regional Geoid determination

Where,

N_{GGM} is long wavelength component for global geopotential model,

$N_{\Delta g}$ is high frequency Geoid signal from local terrestrial gravity data,

N_H is contribution from topographic variabilities,

ϵ_N is random error component,

Δg_F is observed free-air gravity anomaly data,

Δg_{GGM} is gravity signal computed from GGM and

Δg_H is terrain effect on gravity.

Although Geoid undulations are most sensitive from low to medium frequency spectrum of the gravity field, in rough topography, all three data sets are necessary for estimating N.

2.1. THE REMOVE-COMPUTE-RESTORE TECHNIQUE

Eq. $N = N_{GGM} + N_{\Delta g} + N_H + \in_N$ can easily be implemented in most commonly adopted and applied approach to regional gravimetric Geoid determination called Remove-Compute-Restore (RCR) technique In RCR approach the gravity anomalies used in Stokes' integral equation have the contribution of the topography and the GGM removed. Thus, the preprocessing stage involves the computation and removal of the GGM and terrain contributions from the observed gravity anomalies and the post processing step involves the restoration of the GGM and terrain contributions to N.

This scheme satisfies a solution to the geodetic boundary-value problem when formulated for a higher than second-degree reference model of the figure of the earth. In this scheme, the low frequency Geoid undulations generated by GGM (N_{GGM}) are extended into the high frequencies by global integration of terrestrial gravity anomalies $\Delta(g_M)$ containing high frequency Geoid signals, using:

$$N = \Delta N_{\mathrm{GGM}} + \frac{R}{4\pi\lambda} \iint_{\sigma} \Delta g^{o} S(\Psi)\, d\sigma \qquad (2.3)$$

The surface spherical distance ψ between the computation point (ϕ, λ) and the surrounding points (ϕ', λ') on the sphere is given by (Hieskanen& Moritz, 1967)

$$\cos\phi = \sin\phi \sin\phi' + \cos\phi \cos\phi' \cos(\lambda' - \lambda) \qquad (2.4)$$

The performance of RCR can further be enhanced if a modified integration kernel is used in the generalized Stokes' scheme. Studies done by various authors have revealed that application of an appropriate kernel modification approach in RCR method significantly improve the Geoid model results.

2.2. KERNEL MODIFICATIONS

The generalized Stokes' scheme and RCR approach offers the computational convenience in regional Geoid determination besides the fact that the approximation made in such approaches may introduce the truncation error of unknown magnitude. In such circumstances the best way is to reduce the magnitude of truncation error through some mathematical manipulation rather than making trials to completely remove its effect. The recent trends of applying the modified procedure of Geoid computation have shown that the so-called truncation error is not the only error to be taken care of but a proper treatment is required for combined effect of other error sources too.

Therefore, the objective of modifying the Stokes' kernel which works as weight function for gravity anomaly data should be to reduce the combined effect of all the error sources in Geoid computation to a certain level of acceptance for modern geodetic applications such as the determination of orthometric height from the GPS.

2.3. TYPES OF MODIFICATION

The pioneering work of Molodensky and several other authors have proposed different kind of modifications to Stokes' integral. These modifications have been based on different criteria and can be broadly classified as deterministic and stochastic

Various types of deterministic kernel modifications have appeared in geodetic literature. The studies carried out in different parts of world have shown that each kernel behaves differently depending upon the error characteristics of data in each region. The basic criteria of using a particular type of kernel is to obtain a Geoid solution which is best fit to GPS/levelling results as computing the orthometric heights from GPS has been predominantly important application of Geoid in most of the regions. Present study envisages the use of different existing modified kernels and to use them in different frequency band in order to exploit the benefit of each and to arrive at an optimal combined solution for the Geoid. Thus, the technique offers a relative weighting of data to compute the truncation biases separately for each frequency band.

Some of the Kernal Modifications are:

- Wong & Gore Modification
- Meissl's Modification
- Heck &Gruininger's Modification
- Vanicek&Kleusberg's Modification

2.4. HYBRID GEOID MODEL APPROACH

Comparing the gravimetrically-derived and GPS-derived Geoid undulations, over a network of spirit levelled benchmarks has been a standard geodetic practice over the last fifteen years. The RCR approach along with simple kernel modification or band-limited kernel modification may reduce not only truncation error but also errors from other unknown sources. Practically speaking, if purpose of Geoid model determination is to derive orthometric height from GPS, which is true in most of the cases, the model still needs certain corrections to improve the accuracy of fit to GPS/levelling results. The concept of hybrid Geoid model is based on the principle of super imposing a corrector surface over the gravimetric Geoid in the region of interest. There are various method of developing a corrector surface for gravimetric Geoid solution e.g. interpolation with weighted average, similarity transformation (Vella, 2004), Least Square Collocation (Moritz, 1980). Least Square Collocation (LSC) is the most commonly used technique and has been applied in this study a reliable Geoid model may be derived by combining the two sets of data and subsequently correcting the long wavelength errors.

2.5 LEAST SQUARE COLLOCATION (LSC)

LSC is the most general form of least square adjustment process which performs adjustment filtering and prediction steps within a combined algorithm. The statistical approach to collocation is more precisely defined as the determination of a function by fitting an analytical approximation to a certain number of given linear functionals and frequently used in numerical mathematics. Therefore the present technique for determining the Geoidal heights by least-squares prediction using a covariance function is called least square collocation.

The method comprises a number of steps which include:

a. Determining the measurement errors (noise) by filtering.
b. Computing the model parameters by adjustment.
c. Determining the signals and interpolation values in the new point of which the parameters will be derived.
d. Computing the root mean square errors of measurements and derived parameters.

The three types of height i.e. GPS, levelling and gravimetric Geoid are considerably different in terms of physical meaning, reference surface, definition realization, observational method and accuracy, they should fulfill the simple geometrical relationship:

$$N = h_{GPS} - H_{ort} + \in \qquad (4.48)$$

Where, h, H and N have their usual meaning and $\in$ are small quantities due to the deflection of the vertical and the curvature of plumb line.

2.6 CORRECTOR SURFACE FOR GRAVIMETRIC GEOID

The development of corrector surface is basically the problem of modeling the residuals of GPS/leveling (h-H) and gravimetric Geoid heights (N) at co-located points. The model is generally obtained by fitting a plane to residual data, which is defined as the trend surface in LSC process. This removal of trend principally models the long wavelength errors in the global Geoid model and GPS ellipsoidal heights.

2.7 STAGES OF MODELLING

To briefly summarize the methodology of gravimetric Geoid determination and its refinement (hybrid Geoid) which is used in the modelling involves the following stages.

1. Preprocessing of data

- Apply the appropriate corrections (e.g. atmospheric, gravity formulae etc.) to the free-air gravity anomaly data.

$$\Delta g_{cor} = \Delta g_{obs} - \Delta g_{A} - \Delta g_{G} \tag{2.6}$$

- Compute the terrain correction from a given DEM for the test area.

2. Remove steps

- Convert corrected gravity anomalies Δg_{cor} to Fay gravity anomalies Δg_{fay} by applying terrain correction (TC), to account for Helmert's second method of condensation.

$$\Delta g_{fay} = \Delta g_{cor} + TC \tag{2.7}$$

- Remove the contribution from global gravity model to get the residual gravity anomalies.

$$\Delta g_{res} = \Delta g_{fay} - \Delta g_{GGM} \tag{2.8}$$

3. Compute steps

- Grid residual gravity data.

- Apply modified Stokes' scheme with deterministic modified/band limited kernel and compute the residual Geoid height (N_{res}) from a numerical solution of Stokes' integral.

4. Restore steps

- Restore the long wavelength effect of the global geopotential model to the residual undulation of the co-Geoid and add the indirect effect computed from a digital elevation model to get the final Geoid (N).

$$N = N_{res} + N_{GGM} + N_i \quad (2.9)$$

5. Hybrid geoid

- Compute the Geoid differences (DN) between gravimetric Geoid height N and GPS/levelling Geoid height at calocated points.

$$\begin{aligned} DN &= N_{GPS} - N_{Grav} \\ &= (h_{GPS} - H_{level}) - N_{Grav} \quad (2.10) \end{aligned}$$

6. Develop corrector surface by applying LSC approach to Geoid differences (DN) data.
7. Apply corrector surface to Gravimetric Geoid to get hybrid Geoid model.

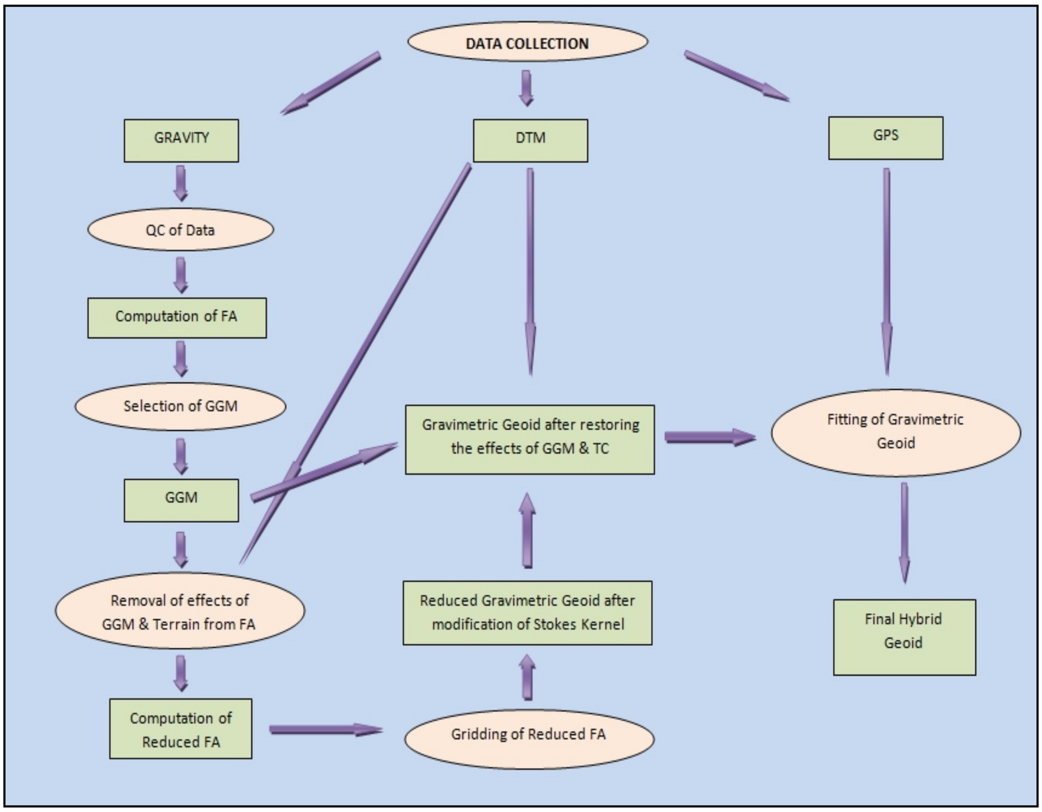


Fig 2.2 Flowchart of Hybrid Geoid Computation

The hybrid Geoid model approach expresses the corrector surface as an analytical surface produced by fitting a multiparameter high degree polynomial. It is further emphasized that the hybrid Geoid model works with better accuracy than the gravimetric Geoid only when there is a sufficiently dense network of GPS/Levelling points is used for structuring the corrector surface. This kind of approach is essentially applicable to only a limited region and might be possible to obtain an acceptable level of accuracy for interpolation points.

CHAPTER 3: TECHNICAL SPECIFICATION

3.1. SPATIAL RESOLUTION

The GEOID model and associated gridded products are being generated as 5 arc-minute grids. The GPS on bench mark dataset is provided at discrete, heterogeneous locations, which are limited by the availability of bench marks and GPS observations.

3.2. GRID CELL VALUES

Geoid heights are reported as 4-byte binary numbers. This implies 10^{-38} precision, but practically, the precision is limited to approximately 1 mm.

3.3. UNITS

Geoid heights are in meters. The estimated uncertainty is provided in meters at 1-sigma (1σ) standard in the grid.

3.4. DATUMS

Indian Geoid including NHP Geoid is intended for use with coordinates in the Indian Terrestrial Reference Frame [epoch 2005.00]. It provides orthometric heights consistent with the India Vertical Datum (VD) of 2009 (IVD 2009).

Indian Geoid does not incorporate any time dependency in the model, which is consistent with the static nature of the VD. Discrepancies in Geoid have been mitigated with updated levelling adjustments, outlier detection schemes, and redundant observations, as permissible.

3.5. COVERAGE AREA

Latest Geoid model developed for National Hydrology Project (NHP) is developed specifically for the project area, as defined in Table 3.1. *It is not recommended for use outside of the land covered portions of these areas and outside the territorial boundaries of India due to insufficient GPS on bench mark constraints and/or lack of jurisdiction*.

The coverage area does include some foreign territories including China, Nepal and Bangla-Desh due to the format of grid generation but it MUST NOT be used for these territories. Survey of India is responsible for the portion of Geoid model falling within the territorial jurisdiction of the Republic of India.

5' resolution	Min. Latitude	Max. Latitude	Min. East Longitude	Max. East Longitude
Parts of Uttarakhand, Uttar Pradesh and Bihar	24 degree	31 degree	77 degree	86 degree

Table 3.1: Geographic Areas for NHP Geoid
(valid within the International Boundary of India)

CHAPTER 4: OBSERVATIONS AND RESULTS

4.1. INPUT DATA FOR GEOID COMPUTATION

The quality of Geoid models is strongly dependent on the input data entering into the computation. The quality and coverage of gravity and DEM data play an utmost importance role, and influence to a greater extent, the accuracy of the resultant Geoid mode1 solution. The theoretical perfectness of any method does not have any impact, if the input data is insufficient or its quality is not up to the mark. It is therefore important to study the data carefully prior to its use in the process of Geoid computation. A proper treatment and pre-processing is invariably required since the gravity and height data are collected with different methods of observations and with equipment of different order of accuracy over a period of several decades.

4.1.1 DEM for the Data Area

Gravity anomalies are dependent on the topographic masses. In order to consider the influence of topographic masses properly, one has to use a reasonably accurate DEM. For convenience of computation the DEM should cover the whole data area. In the absenc e of an officially published local DEM for the data area, global DEM SRTM 30" is considered and tested for their suitability for Geoid modelling process. Figure 4.1 illustrates the 3-D view of topographic mass distribution in the area.

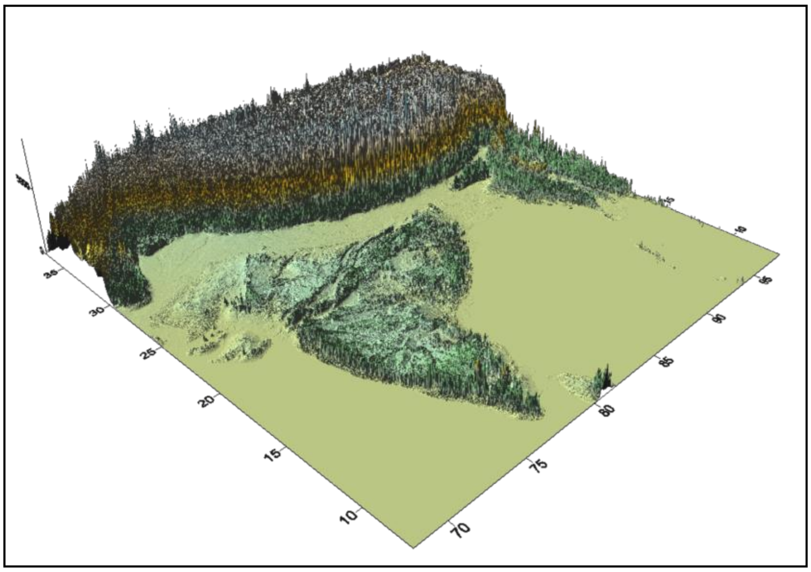


Fig 4.1 3-D Graphical representation of data area.

4.1.2 Gravity Data

Geodetic and Research Branch (G&RB) of Survey of India is the nodal agency for collecting and maintaining gravity data base in India. The data base is created and maintained within the framework of activities G&RB. The gravity observations have been carried out by G&RB as a part of their core activities for more than 100 years. Thus, since 1902, when the systematic gravity observations commenced in India, the process of collection of data has passed through many phases in terms of the order of accuracy of instruments used, procedure of observation and data processing. For each gravity point the information is arranged as: point number, geographical coordinates (latitude and longitude), height (related to MSL observed mainly by barometric method) and the free-air gravity anomalies. The locations of these points are shown in Fig. 4.3& 4.3. For these data points, the normal gravity γ_0 on the reference ellipsoid is computed by the series of expansion defined for the Geodetic Reference System 1980 (GRS-80):

$$\gamma_0 = \gamma_e(1 + 0.0052790414 \sin^2\phi + 0.0000232718 \text{ Sin}^4\phi + 0.0000001262 \text{ Sin}^6\phi + 0.000000000 \text{ Sin}^8\phi \qquad (4.1)$$

Where $\gamma_e = 9.7803267715 \text{ ms}^{-2}$ normal gravity at equator and $\phi$is geodetic latitude.

The distribution of gravity data points is not adequate as seen from the Fig. 4.2. Some of the areas have sufficiently dense network of control points whereas other parts are completely left out. In such a scenario, there is no other alternative but to rely on the accuracy of the predicted data for these areas. Nevertheless, the available data provided a useful source of gravity and terrain information for gravimetric Geoid computations.

Approximately total 11243 Nos. of old existing gravity points were available & Total 2368 Nos. of new gravity points were observed under the project.

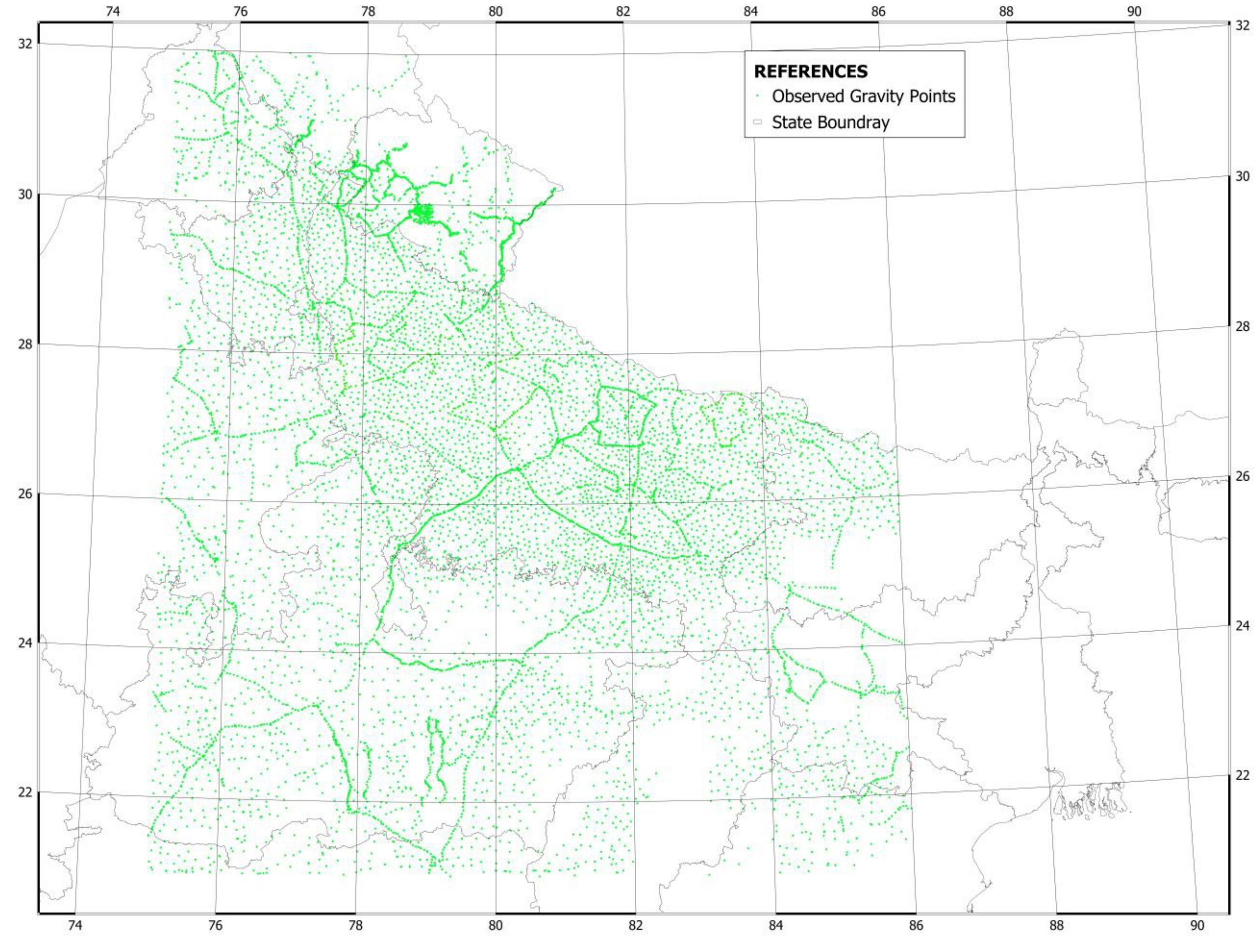


Fig 4.2 Geographical distribution of Gravity data points for NHP

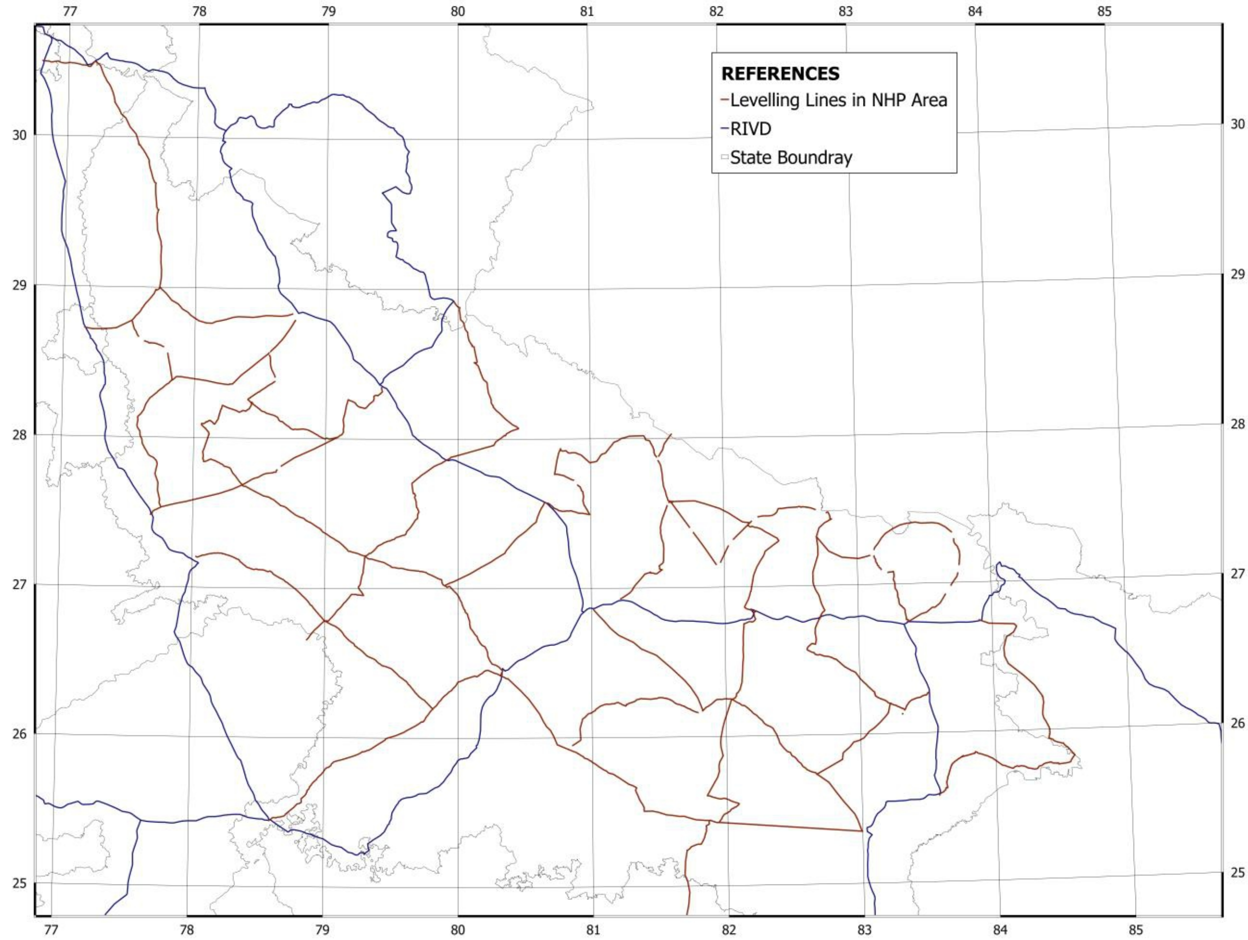


Fig 4.3 Geographical distribution of gravity data points on leveling Lines for NHP

4.1.3 GNSS /Levelling Data

For the gravimetric Geoid model validation the difference between the ellipsoidal (obtained from GNSS measurements) and spirit levelled heights measured at the same point are generally used as a discrete geometrical control. In the absolute verification, the accuracy of gravimetric Geoid is estimated by using GNSS networks that have been tied to International Terrestrial Reference Frame (ITRF) and spirit levelled heights that have been tied to the local vertical datum. One should keep in mind that the local vertical datum does not necessarily coincide with the Geoid for several reasons as already discussed in this report. On the other hand, gravimetric Geoid refers to global reference system.

The observables h, H and N in equation (1.1) contain random noise, datum in consistencies and other possible systematic distortion in the three height data sets (e.g. long wavelength systematic error in N, distortion in the vertical datum due to non adjustment or an over constrained adjustment of the levelling network, deviation between gravimetric Geoid and reference surface of levelling datum). In addition various geodynamic effects (post glacial rebound, plate deformation near subduction zones, sea level rise, monument instabilities) are not completely considered and theoretical approximation in the computation of either H or N (e.g. improper, or nonexistent terrain/density modelling in the Geoid solution, improper evaluation of orthometric height using normal gravity values instead of actual surface gravity observations, neglecting the effect of the Sea Surface Topography (SST) at the tide gauges, error free assumption for the tide gauge observations) are not rigorous enough.

However keeping in mind that the primary application of a Geoid model is the transformation of GNSS heights, the inter-comparison of a Geoid model, GNSS and spirit leveled heights gives an indication of the suitability of the gravimetric Geoid for this purpose.

In the year 2005-2006 a national GNSS network was established by G&RB, Survey of India. The average distance between neighboring points was about 50-70 km. The locations of points were chosen in such a way that it can be connected with the levelling benchmarks of Indian height datum with minimum efforts. The GNSS baselines were observed using dual frequency carrier phase GNSS receivers and Choke-ring antennas. The observation period for national network points was kept minimum 12 hrs with 30 seconds epoch interval considering that the data would be processed in combination with IGS network stations which are more than 1000 km away from the point of observation. Each control point was observed independently in static mode and connected with levelling benchmarks of Indian height datum through the system of fore and back levelling done simultaneously. The standard of accuracy was kept $3\sqrt{K}$ mm (K in km) in both the directions. To provide the ellipsoidal height of the GNSS/levelling points with highest order of accuracy all necessary precautions (e.g. cutoff angle 15°, multipath protection sky clearance etc.) were taken during the observations. The observed data in each case was processed in combination of International GNSS Service (IGS) stations with BERNESE software using precise orbit information. We could conclude that high precision GNSS/levelling data set is reasonably accurate (the combined error of GNSS and levelling cannot be higher than 2-3 centimeters) for the evaluation of quality of the gravimetric Geoid model.

The distribution of GNSS/Levelling points, as seen from Figure 4.3, is not uniform throughout the area and does not provide a good geographical coverage. The data may be just sufficient to evaluate the gravimetric Geoid model accuracy but do not provide the complete information about the probable departure of gravimetric Geoid from the geometric Geoid. For this reason, considering the topographical variations within the target area might be beneficial while using these points for computing the corrector surface. It is already seen from Figure 5.1 that the area consists of moderately undulating terrain which is most likely to follow a systematic trend in Geoid variations. Thus, the available GNSS/Levelling data, whatsoever scanty it may be, must fulfill the requirement of most likely representative of the Geoid –ellipsoid differences in the area. Approximately 12214 linear kms of HP leveling was carried out during the project. Approximately total Nos. of 469 GNSS stations were observed under the project.

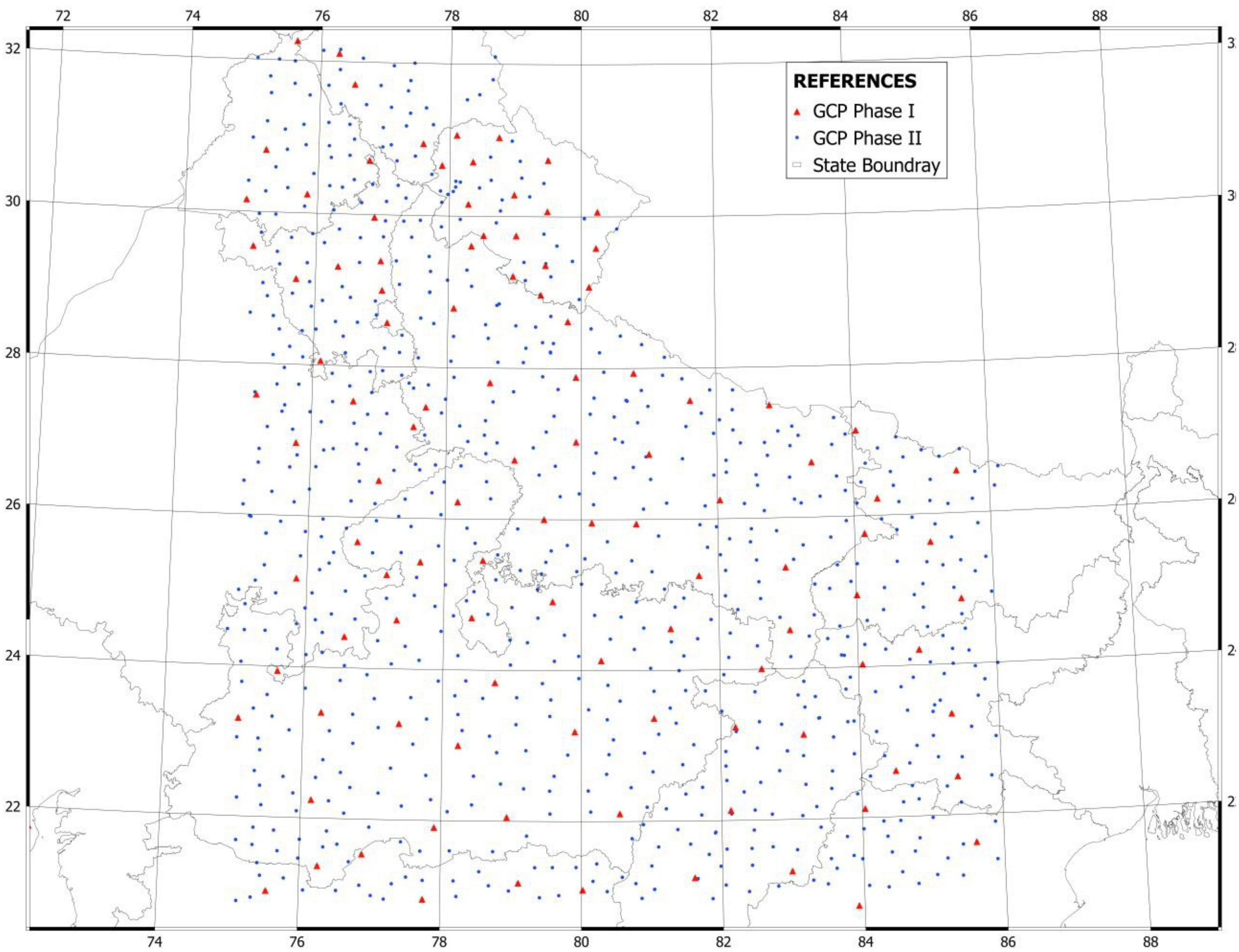


Fig 4.4 Geographical distribution of GCPs for NHP

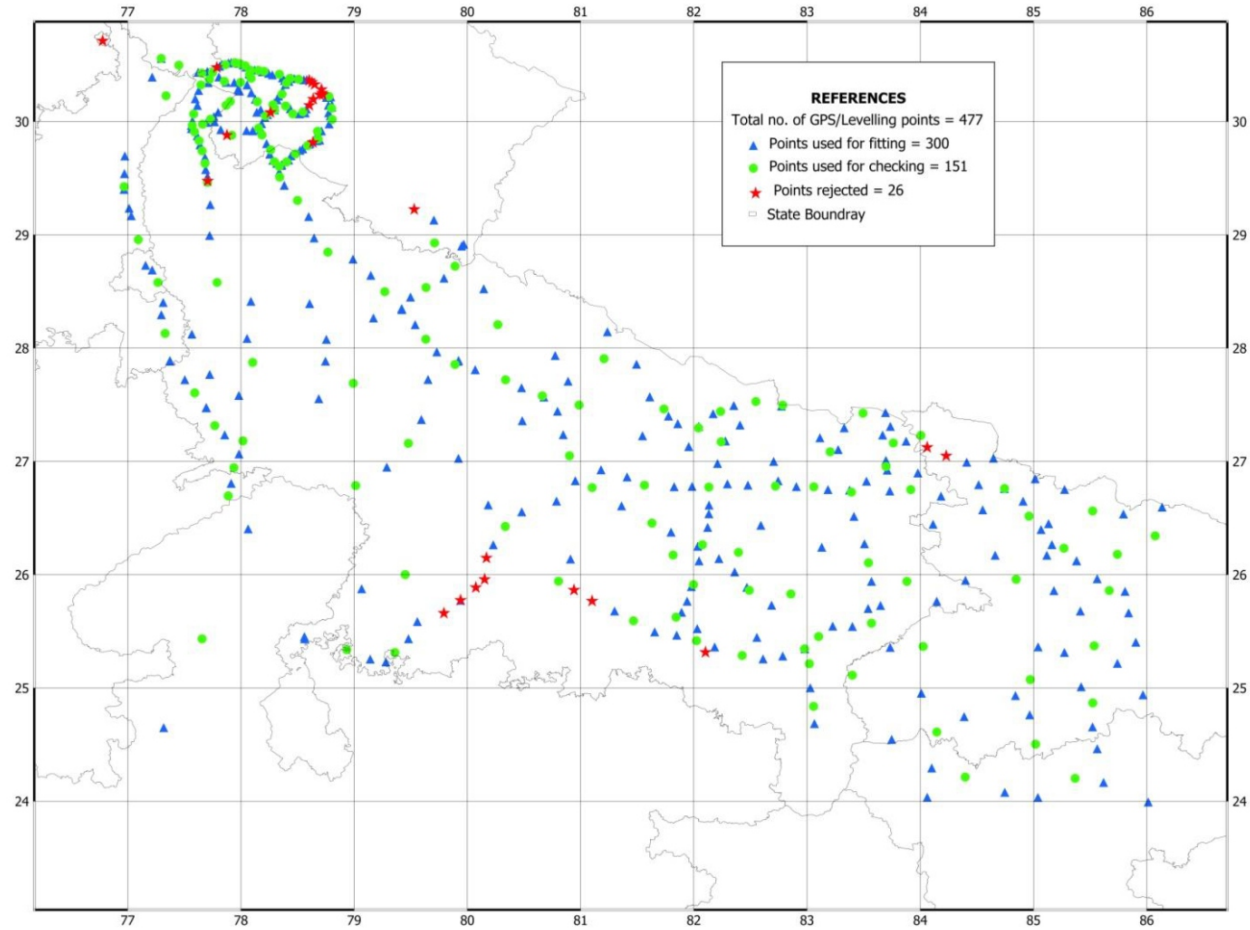


Fig 4.5 Geographical distribution of Points (BMs/GCPs) on which 'N' is known for NHP.

<u>AREA OF MODEL:</u> The aim of this project was to prepare High Resolution Geoid Model for approx. 5, 47,400 sq.kms area coving the state boundary of the state of Uttar Pradesh. The area is bounded by the 24°N to 31°N in Latitude & 77°E to 86°E in Longitude.

The area falls in the Sheet Nos. 53/F, G, H, J, K, L, O, P; 54/D, E, F, G, H, I, J, K, L, M, N, O, P; 55/A, E, I, M; 62/D, H; 63/A, B, C, D, E, F, G, H, I, J, K, M, N, O, P; 64/A, E, I, M; 72/A, B, C; &73/A.

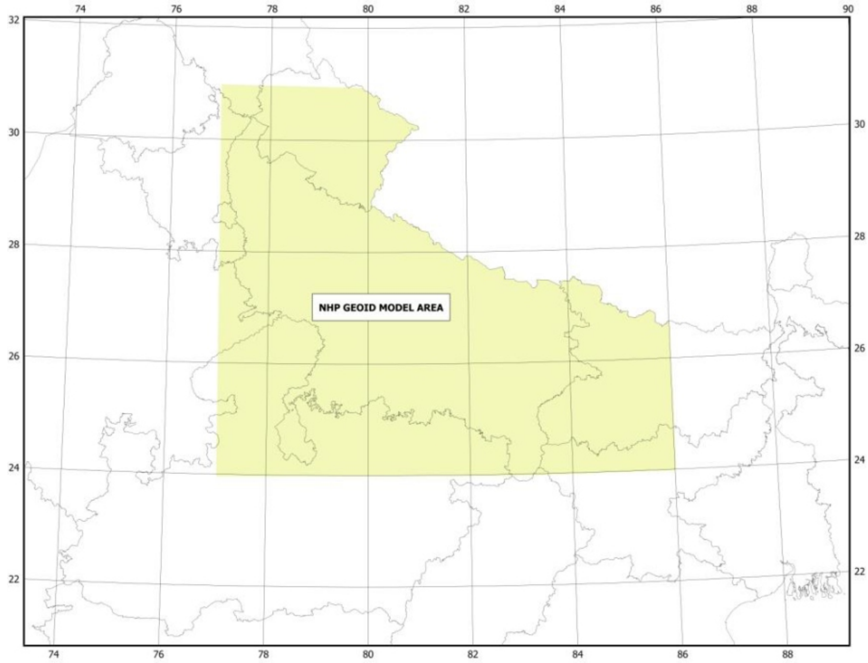


Fig 4.6 NHP Geoid Model Area

4.2. GLOBAL GEOPOTENTIAL MODELS USED

Table4.2 lists the GGMs tested in this study together with the maximum degree of expansion, model type and the source.

Sl. No.	Global Gravity Model	Degree	Data	References
1	**XGM 2016**	719	A, G, S(GOCO05s)	Pail, R. et al, 2017
2	**GOCO05C**	720	A, G, S	Fecher, T. et al, 2016
3	**GGM05C**	360	A, G, S (Goce), S(Grace)	Ries, J. et al, 2016
4	**GECO**	2190	EGM08, S (Goce)	Gilardoni, M. et al, 2016
5	**EGM08**	2190	A, G, S (Grace)	Pavlis, N.K. et al, 2008
6	**EIGEN6C4**	2190	A, G, S (Goce), S(Grace), S(Lageos)	Förste, Christoph et al, 2014
7	**EIGEN6C3STAT**	1949	A, G, S (Goce), S(Grace), S(Lageos)	Förste, C. et al, 2012
8	**SGG-UGM-1**	2159	EGM08, S (Goce)	Liang. W. et al., 2018 & Xu, X et al. (2017)

Table 4.1: Global Geopotential Models tested for NHP

The goodness of fit of GGMs to GNSS/levelling results in the area has been evaluated based upon the statistics given in Table 4.3 and the model for which the RMSE of differences is lowest is selected for use in Geoid modeling process. Seven models namely XGM 2016, GOCO05C, GGM05C, GECO, EGM08, EIGEN6C4, EIGEN6C3STAT& SGG-UGM-1 were checked and GECO was found to be lowest and hence considered for using the reference gravity fields in order to attain the optimized Geoid modeling solution.

Statistics of GGM N & Known N Differences						
Sl. No.	**Global Gravity Model**	**Degree**	**RMSE**	**Mean**	**Min**	**Max**
1	**XGM 2016**	**719**	0.365	-0.329	-0.503	0.095
2	**GOCO05C**	**720**	0.369	-0.317	-0.796	0.130
3	**GGM05C**	**360**	0.377	-0.294	-0.796	0.199
4	**GECO**	**2190**	**0.316**	**-0.288**	**-0.759**	**0.123**
5	**EGM08**	**2190**	0.385	-0.318	-0.767	0.151
6	**EIGEN6C4**	**2190**	0.340	-0.286	-0.752	0.194
7	**EIGEN6C3STAT**	**1949**	0.327	-0.271	-0.740	0.180
8	**SGG-UGM-1**	**2159**	0.318	-0.268	-0.775	0.172

Table 4.2: Fit of Geopotential model to GNSS/levelling Geoid heights for NHP.

Note: Difference = GNSS/Levelling Geoid heights – GGM Geoid heights

4.3 COMPARISON OF GEOID RESULTS WITH GNSS/LEVELLING DATA

For the present study, the GNSS-levelling data set is compared against the Geoid model solution derived using the best estimate of cap size and degree of modification for a particular deterministically modified Stoke’s kernel. The Geoid heights for the location of each GNSS/levelling point each Geoid model solution are interpolated using a bicubic spline interpolation function. The primary aim of this exercise is to select the best fit Geoid model for the target area based on the RMSE of differences between GNSS/levelling Geoid height and heights derived from the gravimetric Geoid. Table 4.3 shows a statistical summary of the differences.

Minimum Diff	Maximum Diff	RMSE	Mean
-0.279	0.338	0.117	0.048

Table 4.3: Statistic of Gravimetric Geoid N & Known N differences in NHP Geoid

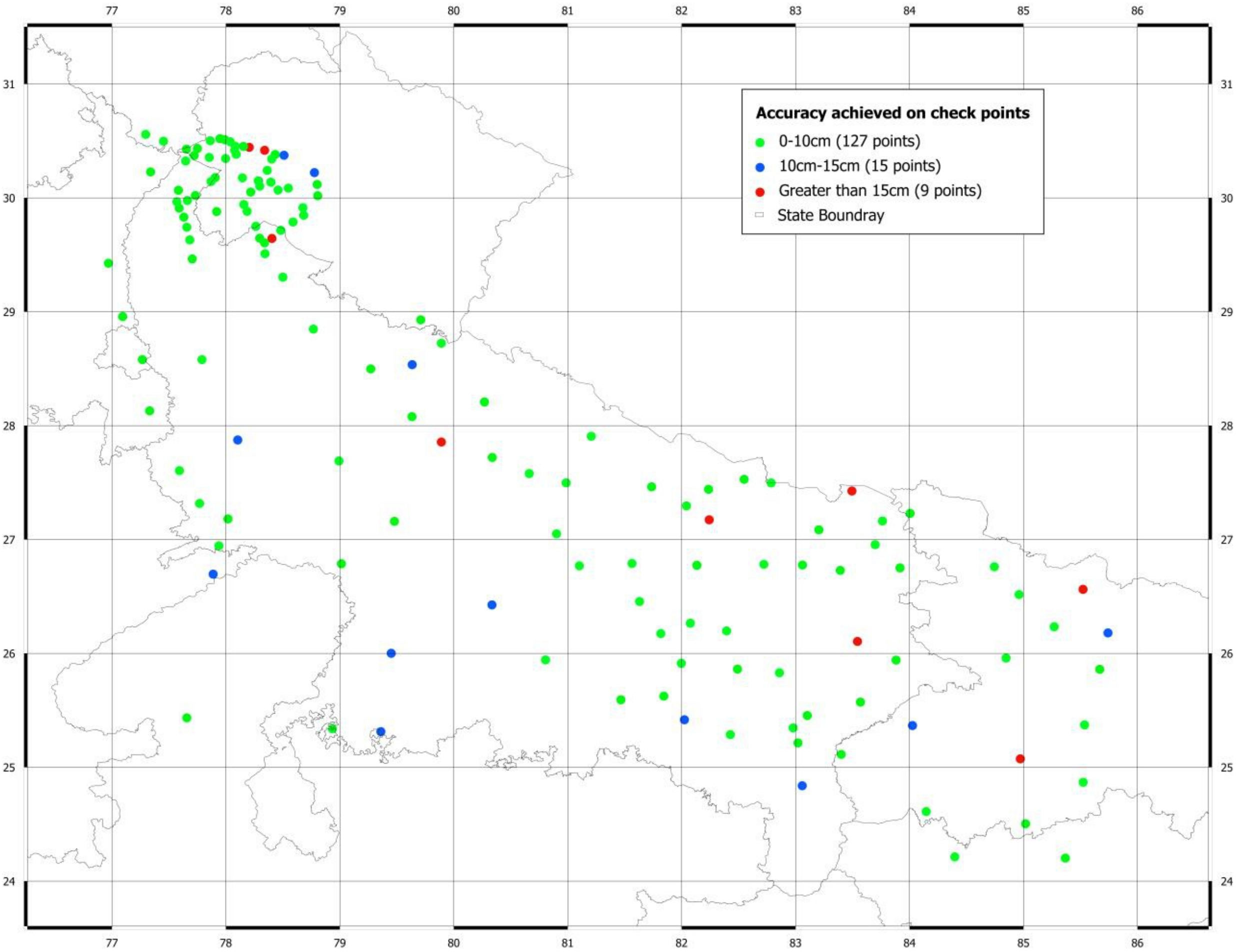


Fig4.7: Distribution of Check Points under NHP.

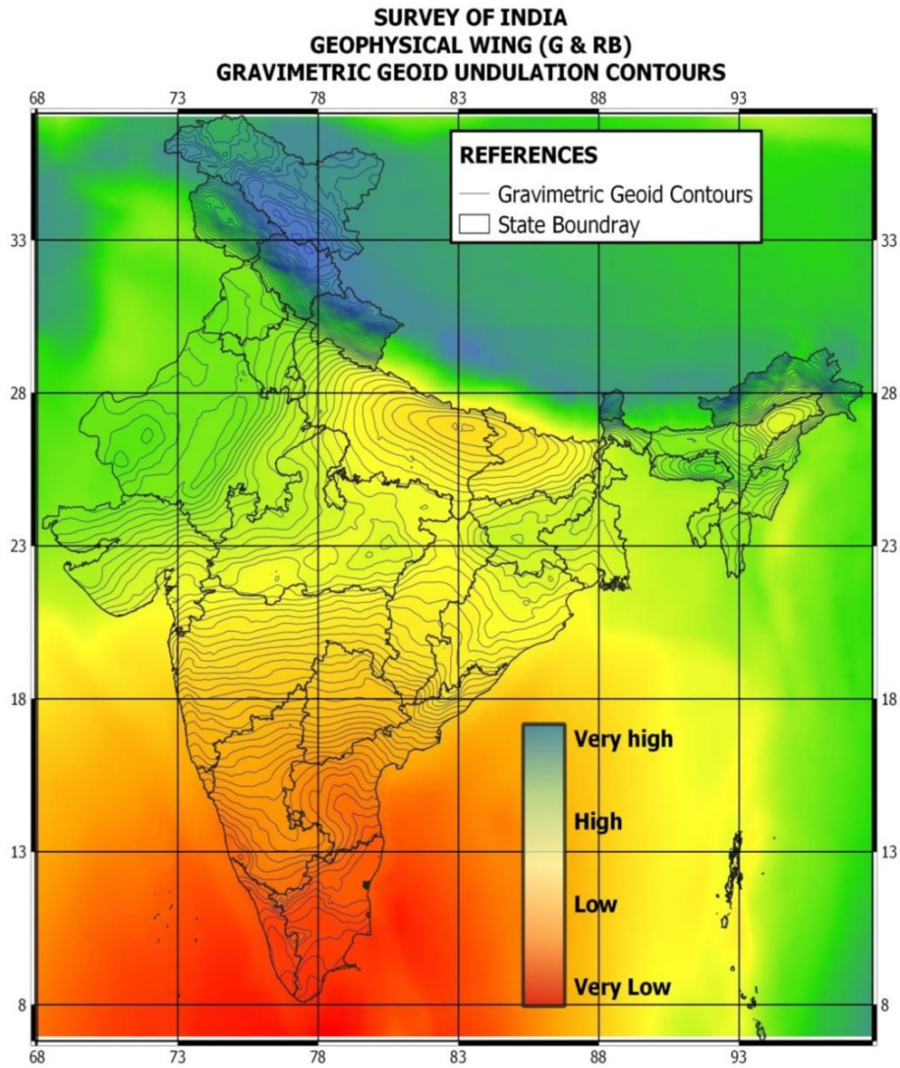


Fig 4.8: Representative Gravimetric Geoid Undulation (Sample)

4.4 COMPUTATION OF HYBRID GEOID

The residuals of gravimetric and geometric (GPS/levelling) Geoid heights are basic data to be used in hybrid Geoid computation. The model is applied to all the discrete GPS/leveling control points and a least square adjustment is performed to estimate the residuals, which are traditionally taken as the final external indication of Geoid accuracy. The adjusted values for the residuals Vj are then geographically modelled in a grid form following the LSC technique. In this process, the residual data have been detrended by fitting the four parameters model and an isotropic Gaussian covariance function has been fitted to the statistics of irregularly distributed residuals after detrending. The LSC formula is then applied for predicting the values of residuals and arranging into a regular grid type structure. From the combinati on of the predicted values of the residuals and the adjusted values for the parameters (x), a connector surface to gravimetric Geoid is finally computed and applied to structure a hybrid Geoid model.

The accuracy achieved on check points (on which 'N' is known) using NHP Geoid has been computed and it is found that when compared to the accuracy achieved on the same check points using IndGeoid, there is an overall improvement in accuracy of about 47%.

Geoid	No of check points	RMSE (m)	Std. Dev. (m)	Mean (m)
NHP Geoid	151	0.077	0.076	-0.009

Table 4.4: Statistics of Hybrid Geoid

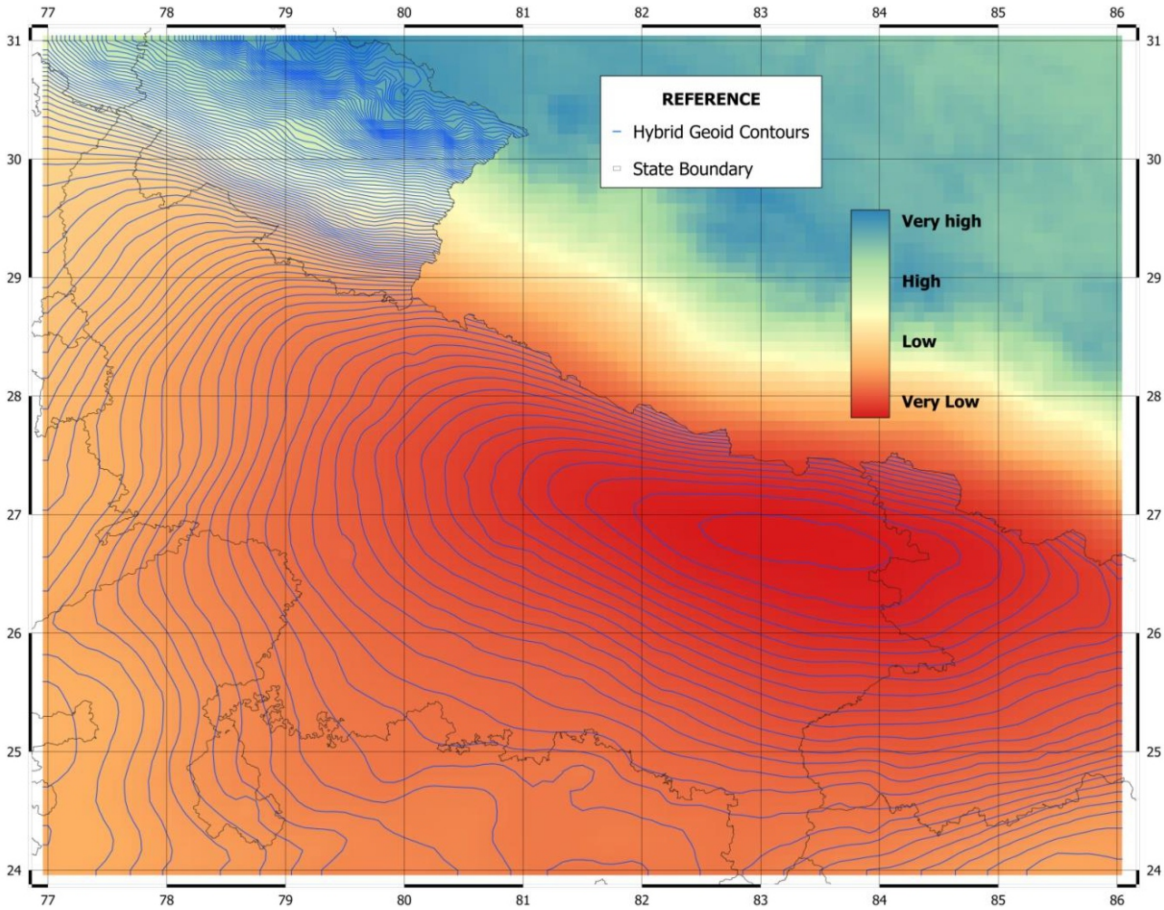


Fig 4.9: NHP Geoid Undulation Contour

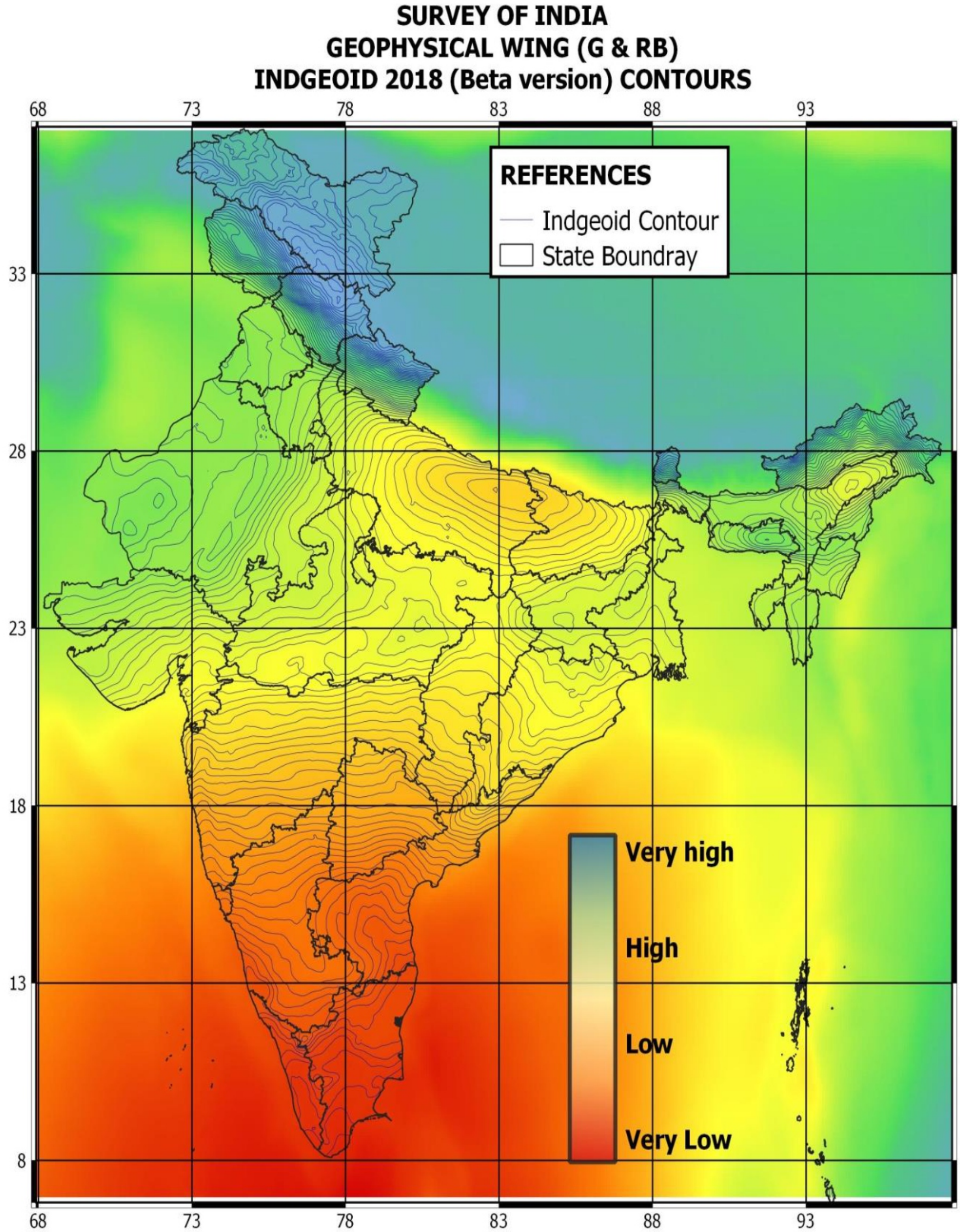


Fig 4.10: INDGEOID Undulation 2018 (Beta version)

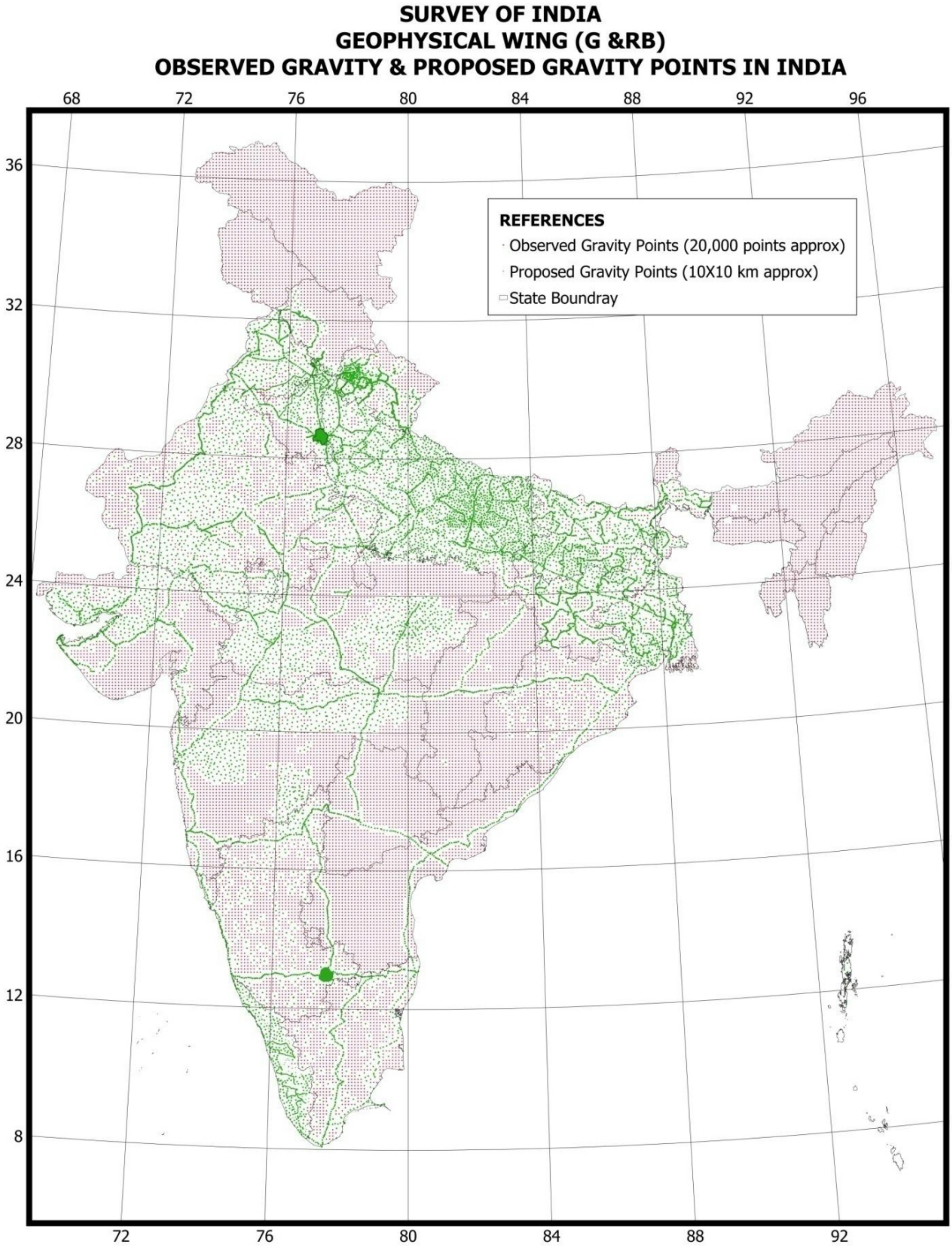


Fig 4.11: Overall Status Present as well as Proposed Gravity Observations

4.5 CONCLUSIONS

From the theoretical Studies and numerical tests carried out in this study, following major conclusions can be drawn

i. Accuracy of Hybrid Geoid prepared evaluated by discrepancies between the hybrid Geoid and GNSS / levelling network points and are within 10 cm (RMSE) accurate within core area of NHP.

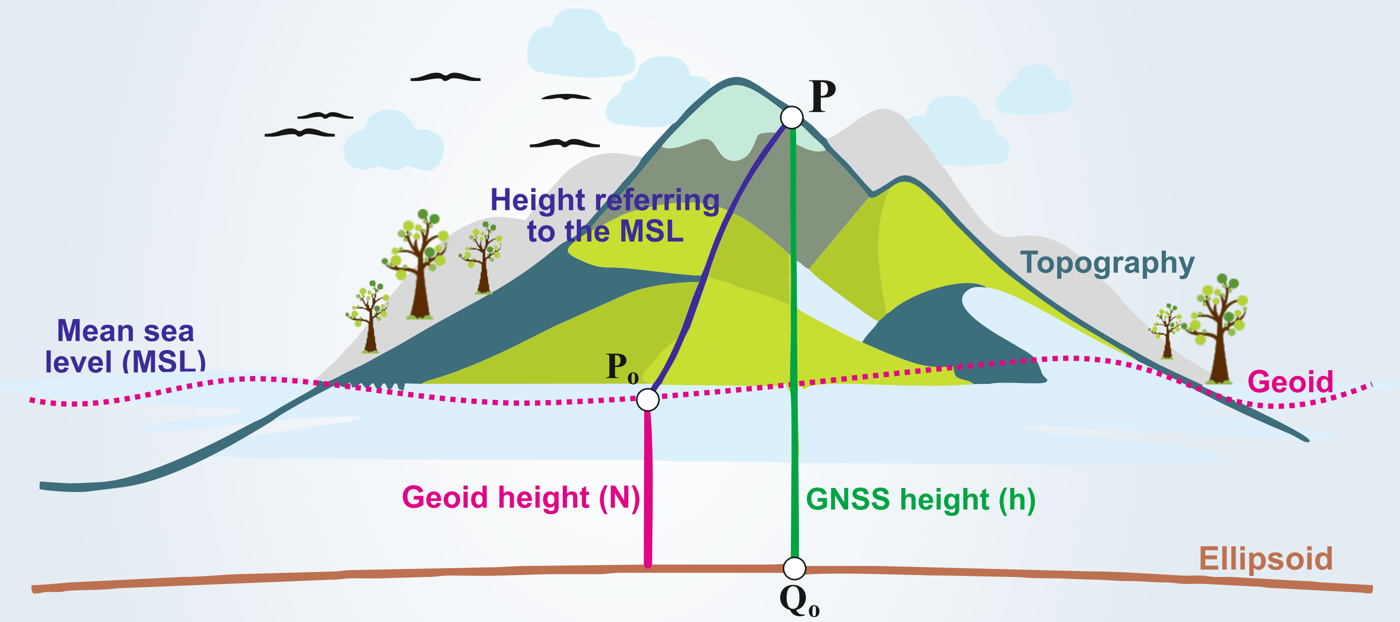
ii. The result of NHP geoid encourages to continue the present execution to fulfill the goal of the development of the Geoid model of the India

4.6 RECOMMENDATIONS

On the basis of the experience derived from this study, following recommendations has been proposed for future work:

i. A nationwide DEM is an immediate necessity for a more sensible gravity terrain reduction for Geoid determination. It is also desirable to have a 3-D digital density model, which may provide the possibility to study the detailed structure of the gravity field. This is particularly useful since the gravity field of India is very complicated due to its vast extent and varying characteristics of rocks beneath. It presents an exceptional case in relation to widely accepted view of gravity field structure. The topography may contain longer wavelength features than the gravity anomalies in some parts of the country. This implies that large density anomalies in some regions may exist below the sub-continent. Geophysicists and geologists across the country may help in finding this information for future work. The critical radius of innermost zone is ($\Delta\psi$) is to be selected very carefully as it contributes a significant part of residual Geoid spectrum. However, its computation is badly affected by insufficient number of gravity data points in surroundings of point of computation. If the resolution of data coverage is poor, the prediction of point within inner zone may add unwanted Geoid signals. This is a complicated situation and only a denser coverage of gravity data can solve the problems. Satellite altimetry data collection, analysis, assessment and implementation of marine Geoid together with the sea surface topography information and marine gravity observations are important for the computation of Indian Geoid. Satellite altimetry and airborne gravity are an alternative to fill up the gaps of terrestrial data, and therefore may be able to provide more detailed gravity field information, which is important for further improvement of the gravity field in terms of coverage, quality and quantity.

ii. A simple inspection of the RMSE of fit of Geoid to the GNSS/levelling data do indicates that results achieved, notably through the 'corrector surface' are on expected line. However, this claim must be balanced against the numbers and suspected quality of the GNSS/levelling data used in the tests. Therefore, in the subsequent studies, a sequential approach based on the findings presented, here would be conducted to ascertain the optimal approach, including some empirical optimization of the integration domain.



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